

# 2022 P5 Exploration Class Selection Test

(Time: 90 min; Full marks: 120 points)

Name: \_\_\_\_\_

Score: \_\_\_\_\_

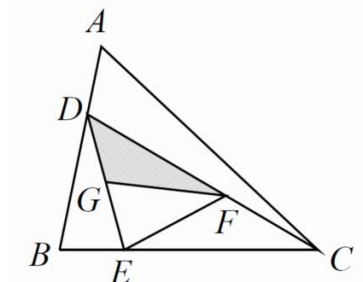
## Part I. Fill in the blanks (4'×8=32' )

1.  $\frac{1}{2 \times 5} + \frac{1}{5 \times 8} + \frac{1}{8 \times 11} + \frac{1}{11 \times 14} + \frac{1}{14 \times 17} = \underline{\hspace{2cm}}$ .

2. Given a 12-digit number 314159265358, if Watson removed six digits to make it a 6-digit number, the maximum value of this 6-digit number is \_\_\_\_\_.

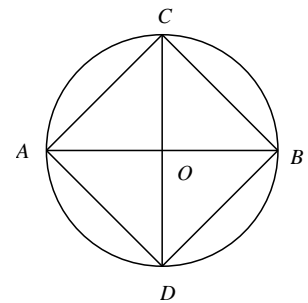
3. At what time between 4 and 5 o'clock will the hour hand and the minute hand of a clock point exactly to the opposite directions? Ans: \_\_\_\_\_.

4. As shown in the figure, the area of the triangle ABC is 24 square centimeters, if  $BD=2DA$ ,  $CE=3EB$ ,  $DF:FC=3:2$  and G is the midpoint of DE, the shaded area of the triangle DFG is \_\_\_\_\_ square centimeters.



5. A 5-digit number  $\overline{8a25b}$  is both a multiple of 8 and a multiple of 9. The value of  $a+b$  is \_\_\_\_\_.

6. As shown in the figure, from A to B, there are \_\_\_\_\_ different paths if a point cannot be passed more than once.



7. Amelia and Brandon are competing in a shooting contest; for each target they hit, they receive 20 points, while for each one they miss, they lose 12 points. With 10 shots for each of them and their combined score is 208, Amelia has a 64-point advantage over Brandon, Brandon hit the target \_\_\_\_\_ times.

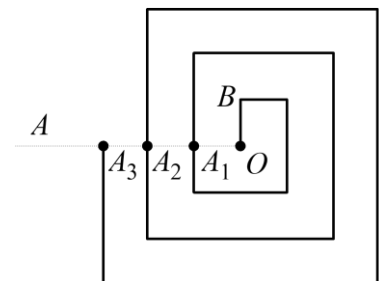
8. Each side of a cube has been divided into 9 squares. If we use the color red, yellow and blue to color these squares such that squares sharing a same side cannot be the same color, the maximum number of square we can color in red is \_\_\_\_\_.

**Part II. Fill in the blanks (6'×8=48' )**

9.  $0.\dot{1}2\dot{3} = \underline{\hspace{2cm}}$ .

(Rewrite this decimal as a proper fraction, where the numerator and denominator are both integers.)

10. The spiral corridor shown in the schematic diagram below has a width of 1. The length of  $OB$  is also 1 and the spiral line intersects with the ray  $OA$  at the point  $A_1, A_2, A_3, \dots$ . If the first spiral starts from  $O$  and ends at point  $A_1$  (with length equal to 7), the second spiral starts from  $A_1$  and ends at point  $A_2$  and vice versa. The length of the tenth spiral would be  $\underline{\hspace{2cm}}$ .



11. There are 1123 pupils sitting a selection test, Amber says: “At least 10 pupils are from the same school”. To make sure that her statement is correct, there are at most  $\underline{\hspace{2cm}}$  schools signed up for this selection test.

12. From 1, 2, 3, 4, 5, 6, 7 and 8, choose five numbers to complete the 1-by-5 grid. Each number can only be used once. If the number in the grey box must be greater than its two neighbors on the left and right, there are  $\underline{\hspace{2cm}}$  possible arrangements.



13. Dividing the 8 numbers 40, 44, 45, 63, 65, 78, 99 and 105 into two groups such that the product of two groups is the same, the sum of the group that contains 99 is \_\_\_\_\_.

14. Of the 60 individuals, 40 play table tennis, 45 play badminton, 48 play volleyball, and 22 play all three sports. At most \_\_\_\_\_ of them do not play any of these three sports.

15. Frank has 10 identical candies, if he eats at least 1 candy everyday starting from today, there are \_\_\_\_\_ ways he can finish all of them.

16. A 3-digit number is divided by 7, 8, 9 and the remainders are written on three different paper respectively. Three intelligent and honest children named Adam, Barbara, and Caitlyn each received one of these papers, and they can only see the information that is on their paper. They gave the following statements, according to the order below.

Adam: "This 3-digit number is not a multiple of 3."

Barbara: "This 3-digit number must be an odd number."

Caitlyn: "I already know what this 3-digit number is, and I can tell you it is a composite number."

The 3-digit number is \_\_\_\_\_.

**Part III. Answer questions (10'×4=40', write the necessary steps, or no points will be given )**

17. In the field of cryptography, plaintext refers to the unencrypted data and cyphertext to the encrypted data. The table below shows the encryption algorithm used to convert the English alphabet into numbers:

<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>	<i>e</i>	<i>f</i>	<i>g</i>	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>
1	2	3	4	5	6	7	8	9	10	11	12	13
<i>n</i>	<i>o</i>	<i>p</i>	<i>q</i>	<i>r</i>	<i>s</i>	<i>t</i>	<i>u</i>	<i>v</i>	<i>w</i>	<i>x</i>	<i>y</i>	<i>z</i>
14	15	16	17	18	19	20	21	22	23	24	25	26

We are given the following formula: When  $1 \leq x \leq 26$ , if  $x$  cannot be divided by 2, then  $x' = \frac{x+1}{2}$ ; if  $x$  can be

divided by 2, then  $x' = \frac{x}{2} + 13$ . If an English alphabet corresponds to a number  $x$ , and by using the formula above, we can turn this number  $x$  into  $x'$ . For example, if the plaintext is a single alphabet “g”, then

$g \rightarrow 7 \rightarrow \frac{7+1}{2} = 4 \rightarrow d$ , and therefore the cyphertext of “g” is “d”.

By using the formula above, what is the cyphertext that corresponds to the plaintext “good”? If the cyphertext is known as “gawqj”, what is the original plaintext?

18. It takes worker A 12 days to finish a task alone. It takes worker B 18 days to finish the same task alone. It takes worker C 24 days to finish the same task alone. If worker A work on the task for a few days before worker B takes over. Worker B then works three times as long as the worker A did before worker C takes over. Worker C then works two times as long as worker B did and the task is finally completed, how many days did it take?

19. A store bought some toys at the cost of \$36 each. If the store sells a toy for \$80, it will sell 50 toys. If every \$2 reduction in price results in the sale of an additional 5 toys, what should be the selling price in order to maximize profit?

20. It is given that the length of the edges of a big cuboid are all natural numbers, if we color all six sides of it in red and cut it into small cubes with edge length 1, there are 12 small cubes with none of its sides red, 32 small cubes with two sides in red, how many small cubes are there with only one side in red?