2017 SMOPS Questions

1. Calculate: $(12345+23451+34512+45123+51234)\div 5$.

2. Given that a, b, c, d are positive numbers and $a<b<c<d$, which of the following expressions has the largest value?

(1) $a\times(b+c+d)$

(2) $b\times(a+c+d)$

(3) $c\times(a+b+d)$

(4) $d\times(a+b+c)$

3. Jason drinks 60% of the water in a bottle and then refills 100 ml. Now the amount of water in the bottle is half of the initial amount. Find the initial volume (in ml) of water in the bottle.

4. Given that a, b, c are prime numbers and $31 + a = 26 + b = 20 + c$. Find the value of $a \times b \times c$.

5. A class of 20 students sits in a circle. They are numbered from 1 to 20 in a clockwise order. Now every third student will leave the circle, starting with students numbered 3, 6, 9 and so on. This process continues until there is only one student remaining. What is the number of the last remaining student?

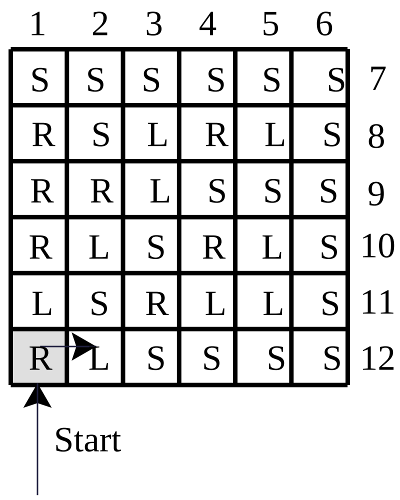
6. The average weight of the people in a group was calculated. Aaron, who weighs 45 kg, joined the group and the average changed to 61 kg. Then Ben, who weighs 71 kg, joined the group after Aaron. The average changed to 62 kg. What was the average weight of the group before Aaron and Ben joined them?

7. A twelve-digit even number $\overline{123A456A789A}$ is divisible by 9 but not divisible by 5. Find the value of A.

8. In the year 2017, Jamie's age is equal to the sum of digits of the year in which she was born. If she was born before the year 2000, how old is she this year?

9. A 3-digit prime number is made of three distinct digits. The unit digit is equal to the sum of the other two digits. Find the unit digit.

10. Bryan enters a 6×6 maze from the shaded cell as shown. Each cell is labelled L for turning left, R for turning right and S for going straight. He moves about the maze according to the label. For example, upon entering the first cell, he turns right, as shown in the diagram below. The process continues until he gets out of the maze. Among the cells labelled 1 to 12, which cell would Bryan exit the maze?



11. It is given that x, y, z are three positive integers smaller than 10. When the product of any two numbers is divided by the third number, the remainder is always 1. Find the value of $x + y + z$.

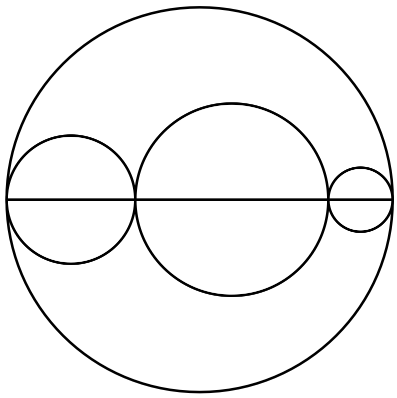
12. In the year 2018, the 1st of January is a Monday. If there are five Mondays in a month, we call it a "Mondayful" month. How many "Mondayful" months are there in 2018?

13. Bryan is the owner of two farms. One day, he places an order to purchase 8 robots for Farm A and 6 robots for Farm B. The robot factory has 14 robots available – 10 in warehouse C and 4 in warehouse D. The shipping cost (per robot) from warehouse to farm is shown in the table below.

Shipping cost (per robot)	To Farm A	To Farm B
From Warehouse C	\$800	\$400
From Warehouse D	\$500	\$300

What is the minimum transportation cost (in dollars) to fulfill Bryan's order?

14. In the diagram below, the circumference of the largest circle is 28. There are three smaller circles with centres lying on the diameter of the largest circle. The circles touch one another as shown in the diagram below. Find the sum of the circumference of the three smaller circles. $\left(\pi = \frac{22}{7}\right)$



15. If x, y, z are all positive integers, find the number of solutions to the equation

$$x + y + z = 7$$

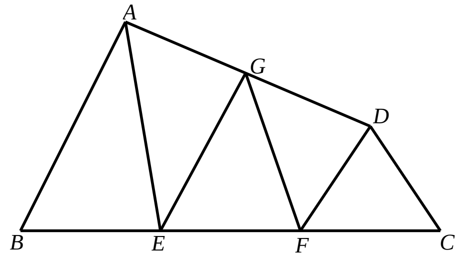
Note that order of unknowns is important. For example, （$x = 1$， $y = 5$， $z = 1$） and （$x = 1$， $y = 5$， $z = 1$） are considered two different solutions.

16. There are 64 small wooden cubes with side length 1 cm. 34 cubes have all six faces painted white while the other 30 cubes have all faces painted black. A $4 \times 4 \times 4$ (in cm) large cube is formed using all the small cubes. What is the least possible area (in 2 cm) of the black region on the surface of the large cube?

17. Using letters a, a, b, b, c, c how many ways are there to fill out a 3×2 table (as shown in the diagram below) such that there are no identical letters on each row or each column?

18. There were 20 different toys in a box. Three children took turns to take the toys out to play and returned them back to the box. It is given that the children played with 11, 17 and 16 toys respectively. What is the least possible number of toys that were played by all three children?

19. In the diagram below, $ABCD$ is a quadrilateral. Points E and F lie on BC such that $BE = EF = FC$. Point G is on AD such that GE is parallel to AB, and FG is parallel to CD. If the area of ΔAEG is 9, find the area of the four-sided figure $AEFD$.



20. What is the least number of cuts needed to cut a piece of 5×7 paper into 35 pieces of 1×1 unit squares? Assume that stacking is allowed when cutting the paper.

21. In the following expression, each letter in *A, B, C, D, E, F, G, H, I, J* represents a distinct digit.

$$A + \overline{BC} + \overline{DEF} + \overline{GHIJ} = X$$

Among all possible values of the sum *X*, which value is the closest to 2010?

($\overline{BC}$ denotes a 2-digit number where the tens digit is *B* and the unit digit is *C*.)

22. In a 12-hours duration, how many times do the hour and minute hand form an angle of $30°$?

23. Three pirates had a gambling session on an island. Before the session, their money was in the ratio $7:6:5$. After the game, their money was in the ratio $6:5:4$ (in the same order). If one of them won 8 dollars, how much money (in dollars) did they have in total?

24. A rabbit and a tortoise entered a 10000 metres race. The rabbit runs at a speed 5 times as fast as the tortoise. They started at the same instant from the starting line. During the race, the rabbit took a nap while the tortoise ran continuously. When the rabbit woke up, he was 4000 metres behind the tortoise. In the end, the rabbit was 200 metres away from the finishing line when the tortoise won the race. What is the distance (in metres) covered by the tortoise while the rabbit was sleeping?

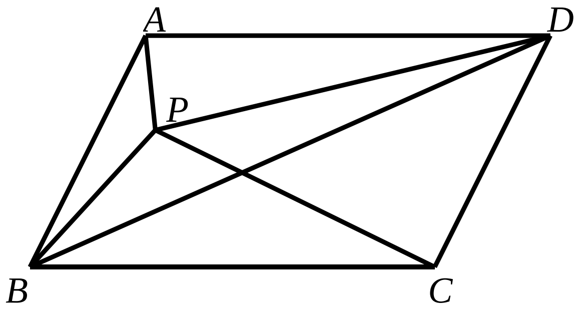
25. Many fractions can be expressed as recurring decimals.

For example, $\frac{26}{111} = 0.234234234... = 0.\dot{2}3\dot{4}$

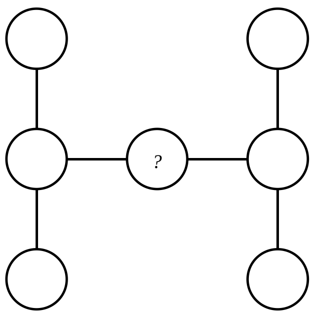
It is known that $\frac{1}{19}$ can be expressed as a recurring decimal $0.\dot{a}_1 a_2 a_3 ... a_{17} \dot{a}_{18}$.

Find the value of $a_1 + a_2 + a_3 + ... + a_{17} + a_{18}$.

26. In the diagram below, point P is inside parallelogram $ABCD$. If the area of ΔABP and ΔBPC is 75 and 124 respectively, find the area of ΔBPD.



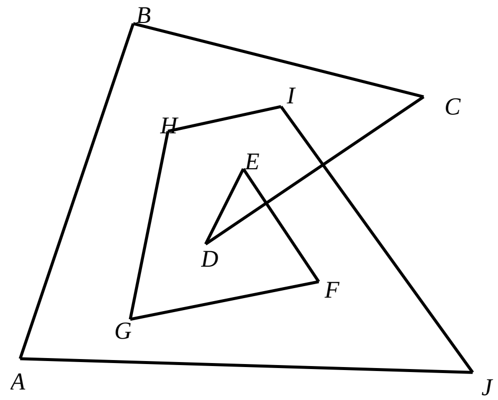
27. Using seven distinct integers from 1 to 9 to fill out the circles in the diagram below such that the products of the three numbers along each straight line are equal, which number is in the centre circle?



28. In the diagram below, find the sum of angles

$$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G + \angle H + \angle I + \angle J$$

in degrees.



29. What is the least number of integers to be randomly selected from

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

to ensure that among the selected integers, there is one number divisible by another number?

30. In the following number formation, if 7102 is the n^{th} number in the m^{th} row, find the value of $m + n$.

$$\begin{array}{ccccccccc} & & & & 2 & & & & \\ & & & 4 & 6 & 8 & & & \\ & & 10 & 12 & 14 & 16 & 18 & & \\ & 20 & 22 & 24 & 26 & 28 & 30 & 32 & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array}$$

2019 SMOPS Questions

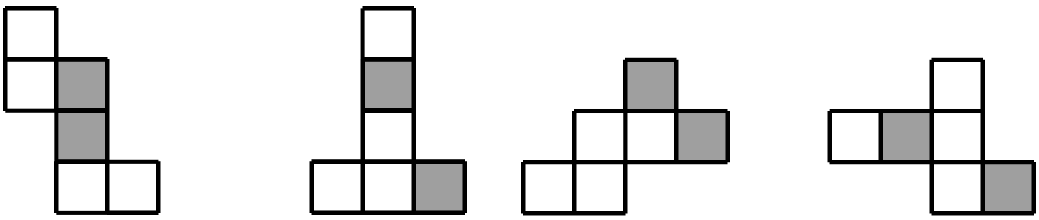
1. If $\overline{ab}+\overline{ba}+b=\overline{aab}$, find $a+b$.
($\overline{ab}$ denotes a 2-digit number where the tens digit is a and the unit digit is b.)

2. Given a and b are positive integers (whole numbers excluding 0), how many ordered pairs of numbers (a, b) are there such that

$$a^2+b^2\le 14$$

For example, (1, 2) and (2, 1) are considered two different ordered pairs.

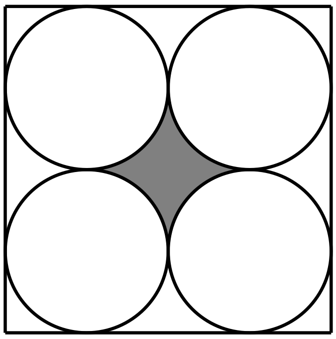
3. When these nets are folded to make cubes, which (if any) will have the two shaded faces directly opposite each other?



A ***B*** ***C*** ***D***

E. none of the given nets

4. In the diagram below, four identical circles are drawn inside a square of area 1600 cm^2. Find the area of the shaded region. Take $\pi = 3.14$.



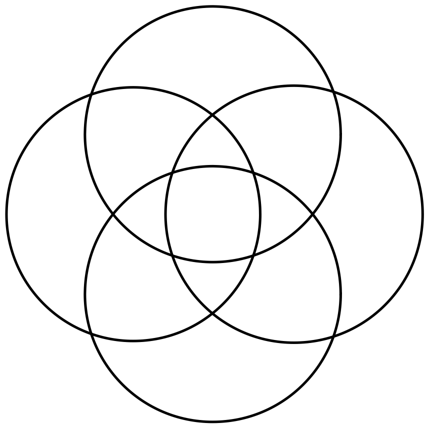
5. In a football tournament, there are k teams. Each team plays against every other team exactly once. Three points are awarded to a team for a win; two points for each of the two teams in a draw; one point for a loss. At the end of the tournament, the total points of all the teams are 24. Find the value of k.

6. Given

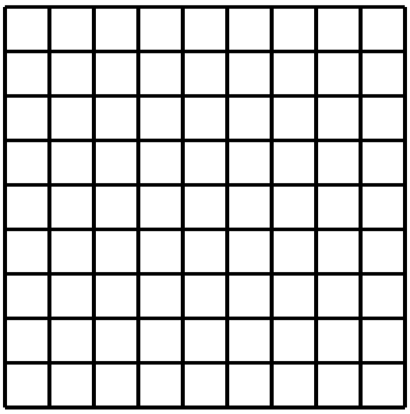
$$\frac{79}{137} = a - \cfrac{1}{b + \cfrac{1}{c - \cfrac{1}{d + \cfrac{1}{e}}}}$$

where a, b, c, d, e are whole numbers, find the value of $a + b + c + d + e$.

7. In the diagram below, each of the integers from 1 to 13 is filled in one of the regions created by the four circles. Each circle contains seven numbers.
The sum of numbers in each circle is S_1, S_2, S_3, S_4
Find the largest value of $S_1 + S_2 + S_3 + S_4$



8. In a 9×9 square grid, exactly 29 of the unit squares are shaded. Other cells are white. All the rows and columns have at least 1 unit-square shaded each. There are x columns and y rows with at least five shaded cells each. Find the largest value of $x + y$.



9. In a test, there are ten multiple choice questions. Four points are awarded for a correct answer. One point is deducted from the total for every wrong answer. No point is given for an unanswered question. How many different total points can the students score in this test?

10. Consider the table below where the numbers are arranged according to a pattern:

	1	3	5	7
15	13	11	9	
	17	19	21	23
31	29	27	25	
	…	…	…	…

The number 9 is in row 2 and column 4.
If the number 2019 is in row m and column n, find the value of $m+n$.

11. A number N is divisible by each of the integers 2, 3, 4, 5, 6, 8 and 9. N gives a remainder of 5 when divided by 7. Find the smallest value of N.

12. On a circular track of circumference 20 m, a robot A travels anticlockwise at a constant speed of 3.5 m/s, while another robot B travels clockwise at a constant speed of 1.5 m/s. They both start at the same point and at the same time. At most how many different points on the track will the two robots pass each other?

13. Find the sum of digits in the number

$$12345678910111213 \ldots 99989999.$$

14. A bag contains blue, white and red marbles. The number of blue marbles is at least equal to half the number of white marbles, and at most equal to one third of the red marbles. Given that the sum of the white marbles and the blue marbles is at least 55. At least how many red marbles are there?

15. Arrange the three numbers below from the smallest to the largest:

$$x=\frac{1}{2}\times\frac{3}{4}\times\frac{5}{6}\times\cdots\times\frac{47}{48}\qquad y=\frac{2}{3}\times\frac{4}{5}\times\frac{6}{7}\times\cdots\times\frac{48}{49}\qquad z=\frac{1}{7}$$

16. Find the 2019$^{\text{th}}$ digit in the number

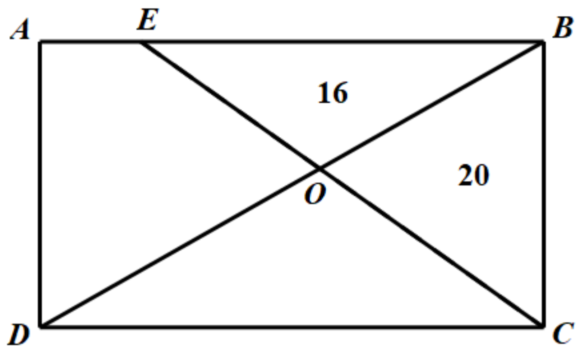
$$12345678910111213\ldots998999.$$

17. In how many ways can we choose two numbers from

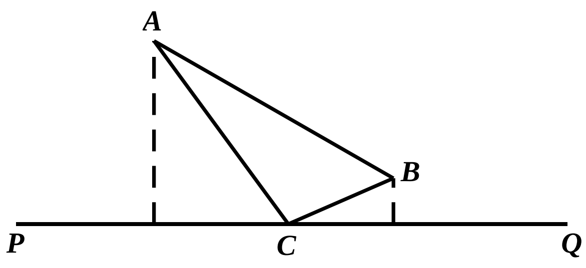
19, 20, 21, 22, … ,78, 79

such that the sum of the two numbers is even?

18. In a rectangle $ABCD$, the areas of two triangles are given. If $AE = \frac{1}{5}AB$, find the area of quadrilateral $ADOE$.



19. In the diagram below, $AB = 3$ m and $PQ = 8$ m. The perpendicular distances from A, B to the line PQ are 2 m and $\frac{1}{2}$ m respectively. A point C varies on the line PQ. Find the largest value of $AC - CB$.

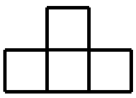


20. Among the integers: 1, 2, 3, … , 49, 50, what is the maximum number of integers that can be selected such that the sum of any two selected numbers is not divisible by 7?

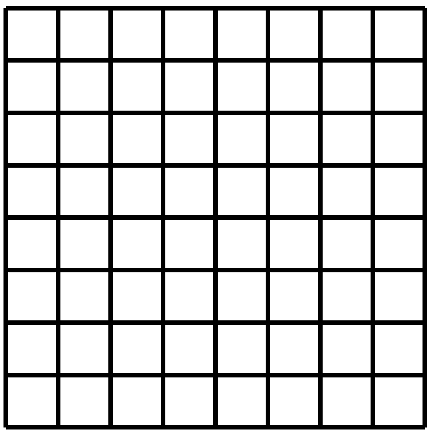
21. Evaluate $\dfrac{\left(2^2+4^2+6^2+\cdots+100^2\right)-\left(1^2+3^2+5^2+\cdots+99^2\right)}{50}$

.

22. Given an ordinary 8 by 8 square chessboard as shown, find the number of different ways of choosing one piece of



which is made up of four square-units.



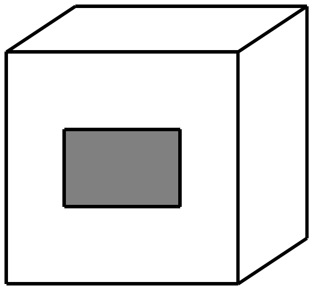
2022 SMOPS Questions

1. Jason drinks 70% of the water in a bottle and then refills 100 ml. Now the amount of water in the bottle is half of the initial amount. Find the initial volume (in ml) of water in the bottle.

2. There are 30 marbles in a bag, comprising 10 of each colour, red, yellow and green. Each red, yellow and green marble weighs 4 grams, 5 grams and 6 grams respectively. Eight marbles are now selected randomly from the bag, with a total mass of 39 grams. What is the maximum possible number of red marbles selected?

3. A list of numbers are defined as follows. The first number is 2000, the second number is 2022. From the third number onwards, every number is the average of the two preceding numbers. What is the integer part of the 15^{th} number? (For example, the integer part of 2.65 is 2.)

4. The diagram shows a cube with side length 6 cm. If a rectangular tunnel with dimensions 2cm by 3cm is made in the middle of the cube, find the amount of increase in the total surface area of the resulting solid in cm^2.

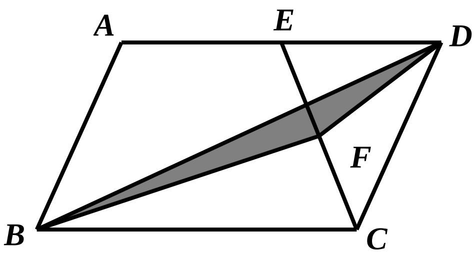


5. Three chess pieces, each of the colour red, black and white, are to be placed on a 6×6 chess board. If any two of the three pieces cannot be placed in the same row or the same column, how many ways are there to place the three chess pieces?

6. A logistic company is tasked to transport 89 tonnes of cargo. The capacity of lorry and caravan is 7 tonnes and 4 tonnes respectively. If each lorry consumes 14 litres of gasoline for the trip while each caravan only uses 9 litres, what is the least total gas consumption (in litres) to complete the task?

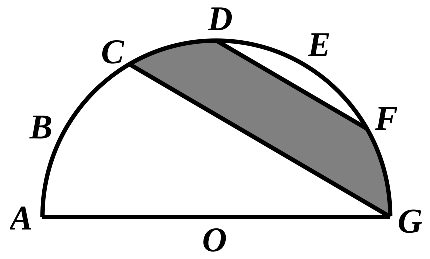
7. Alan and Betty started running towards each other at the same instant, from cities A and B respectively. The ratio of Alan's speed to Betty's is 3:2. Given that they meet at a point that is 18km away from the midpoint of AB, find the distance between A and B.

8. The diagram shows a parallelogram *ABCD*. *E* is the midpoint of *AD*. *F* is the midpoint of *EC*. If the area of the triangle *BFD* is 12 cm^2, find the area of the parallelogram *ABCD* in cm^2.



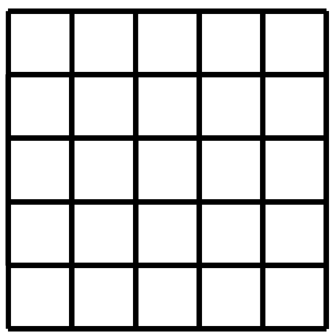
9. The sum of 49 distinct positive integers is 2022. If there are *n* even numbers among the 49 integers, what is the least possible value of *n*?

10. A semi-circle with diameter AG is shown in the diagram below. The entire arc of the semi-circle is divided into 6 equal parts by points B, C, D, E and F. DF and CG are straight lines. Given that the area of the semi-circle is 84cm^2, find the area of the shaded region in cm^2.



11. Given four prime numbers a, b, c and d. If the product of $a \times b \times c \times d$ is the sum of 77 consecutive positive integers, find the smallest possible value of $a + b + c + d$.

12. The diagram shows a 5×5 square with 25 unit squares. Find the least number of unit squares to be shaded such that any 3×3 square in the diagram contains exactly four shaded unit squares.



13. There are five cards A, K, Q, J, 10 and five envelopes labelled ‘A’, ‘K’, ‘Q’, ‘J’ and ‘10’. In the dark, Amy randomly inserts one card into every envelope. How many different ways are there such that every card is in the wrong envelope?

14. Find the smallest multiple of 35 that ends with ‘35’ and has a sum of digits equal to 35. (If your answer is more than 5 digits, shade the first 5 digits. For example, if the answer is 1234535, please shade 12345.)

15. In the following number formation, if 2022 is the n^{th} number in the m^{th} row, find the value of $m+n$.

$$\begin{array}{ccccccccc}
 & & & & 2 & & & & \\
 & & & 4 & 6 & 8 & & & \\
 & & 10 & 12 & 14 & 16 & 18 & & \\
 & 20 & 22 & 24 & 26 & 28 & 30 & 32 & \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots
\end{array}$$

2022 SMOPS Simulation Question

1. Calculate： $28 \times 5 + 2 \times 4 \times 35 + 21 \times 20 + 14 \times 40 + 8 \times 62$.

2. Choose three numbers from six numbers $1\frac{1}{4}$, 1.1, $\frac{7}{4}$, $1\frac{2}{3}$, $\frac{11}{12}$, $\frac{6}{5}$ and mark them A, B and C. The three numbers should make the value of $A \times (B - C)$ as large as possible. Write down the simplest fractional representation of $A \times (B - C)$.

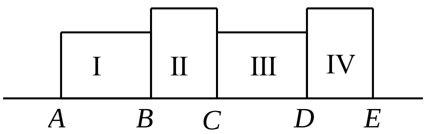
3. Tom practiced handwriting during the summer vacation. He practiced 3 pages a day. 5 days later, he sped up and practiced 20% of all the pages. Then there was 25 pages left. How many pages in total should Tom practice?

4. If a and b are all prime numbers and $3a+7b=41$, then $a+b=$ ___________ .

5. The sum of three different natural numbers is 2001. Divide them by 19, 23 and 31 respectively, the quotients and remainders are all the same. The three numbers are ___________ , ___________ , ___________ .

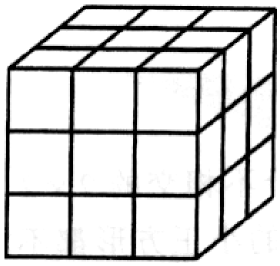
6. It's between 8 o'clock and 9 o'clock now. The hour hand and the minute hand are on both sides of "8", and the rays formed by the hour hand and minute hand are the same distance from "8". What's the time now?

7. As the figure shown, there's a rectangle I with its length 4cm and width 3cm on the line, and its diagonal line is 5 cm. Rotate the rectangle clockwise around the vertex B to reach the position of rectangle II. Continue to rotate it until vertex A reaches the position of E. Find the distance traveled by vertex A. (Take $\pi=3.14$)

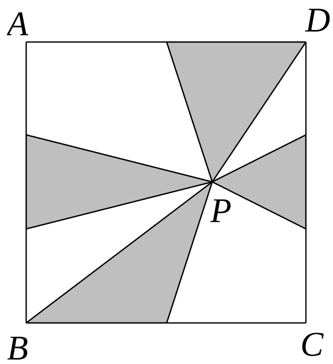


8. Use 100 yuan to by several meal tickets which are worth 2 yuan, 4 yuan or 8 yuan. How many different buying methods are there?

9. As the figure shown, divide each of the six surfaces of the cube into 9 small squares. Use the colour red, yellow and blue to dye the small squares, those with common edges should not be dyed the same colour. How many squares at most can be dyed red?



10. Choose a point P at random in the square $ABCD$ whose side length is 6cm. Bisect one set of opposite sides of the square and trisect the other set of opposite sides and connect them with point P respectively. Find the area of the shaded parts.

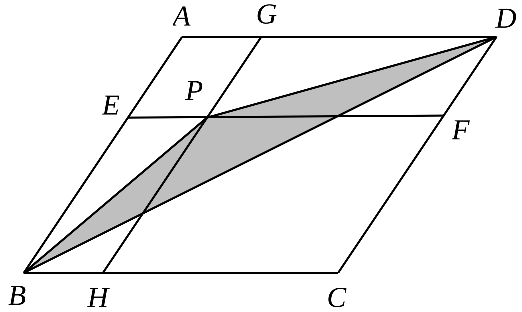


11. As the figure shown, in an addition formula. Each of the 8 letters $QHFZLBDX$ represents a number from 0 to 9. Different letters represent different numbers. If the formula is true, what is the largest value of the four-digit number $\overline{QHFZ}$?

$$\begin{array}{rrrrr} & 2 & 0 & 0 & 9 \\ & Q & H & F & Z \\ + & Q & H & L & B \\ \hline 1 & Q & H & D & X \end{array}$$

12. After writing pure recurring decimals $0.\dot{a}b\dot{c}$ as a simple fraction, the sum of the numerator and the denominator is 58. The three-digit number $\overline{abc}$ = _________ .

13. As the figure shown, draw *EF* and *GH* which are parrallel with the sides of parallelogram through the point *P* which is inside *ABCD*. If the area of $\triangle PCD$ is $8dm^2$, find the difference between the area of parallelogram *PHCF* and *PGAE*.



14. On Teyvat continent, the distance between place A and B is 2022 metres (abbreviated as m). Lisa asks Messenger1 to send letters from A to B with a speed of 1m/s. After one minute, she asks Messenger2 to send letters from A to B with a speed of 2m/s $\cdots$ After k minutes, she asks Messengerk+1 to send letters from A to B with a speed of $(k+1)m/s$. All of the messengers moves at a constant speed. Which messengers can reach place B at the same time? Give all the answers.

15. As shown in the figure, fill in the circles with 13 natural numbers (One number in one circle, numbers in different circles can be the same, 0 is allowed) to let the sums of the numbers on the 9 lines in the figure are different from each other. What's the least sum of the 13 numbers? Write down your proving process.