

2022 P4 Exploration Class Selection Test

(Time: 90 min; Full marks: 120 points)

Name: _____ .

Score: _____ .

Part I. Fill in the blanks ($4' \times 8=32'$)

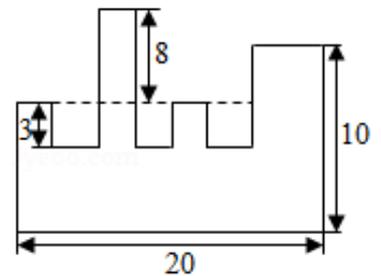
1. Given the 9 by 9 grid below, the sum of the three numbers in every row, column and diagonal must be equal. Then the “☆” corresponds to _____ .

	7	4
3		☆

【解析】如图， $A: 7+4-3=8$ ， $B: (8+4) \div 2=6$ ，所以“☆”代表的数为 $6 \times 2 - 3 = 9$.

	7	4
3	B	☆
A		

2. The perimeter of the following shape is _____ .



【解析】

$$\begin{aligned} & (20+10) \times 2 + 8 \times 2 + 3 \times 6 \\ &= 60 + 16 + 18 \\ &= 94 \end{aligned}$$

3. $\overline{a42b}$ is a multiple of 18. The maximum value of $\overline{a42b}$ is _____ .

【解析】 $18 = 2 \times 9$ ，则 b 是偶数，且 $a+b+4+2$ 的和是 9 的倍数.

$a+b$ 等于 3 或 12

考虑 a 尽可能大，则 $a+b$ 等于 12， b 为 4， a 为 8，此时 $\overline{a42b}$ 的最大值是 8424.

4. A car is travelling from point A to B with speed 30km/h. 4 hours after its departure, a train begins travelling from point A to B with triple the speed of the car. The train catches up with the car at the mid-point of A and B, the distance between A and B is _____ km.

【解析】 $30 \times 4 = 120$ (千米)

$120 \div (30 \times 3 - 30) \times 2 = 4$ (小时)

$30 \times 3 \times 4 = 360$ (千米)

5. A class had 58 participants who registered for extracurricular activities. A total of 32 of them joined the literature society, 24 the art society, and 30 the music society. Thirteen students are members of both the literature society and the art society, twelve are members of both the music society and the art society, and eleven are members of both the literature society and the music society. _____ people joined all three societies.

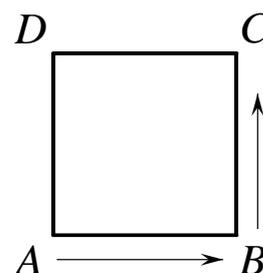
$32 + 24 + 30 = 86$ (人)

$13 + 12 + 11 = 36$ (人)

$58 + 36 - 86 = 8$ (人)

三项都参加的有 8 人.

6. In the square with a side of 90 meters shown below, Alex and Bob leave points A and B, respectively, at the same moment and move anticlockwise. Given that Alex moves at a speed of 63 m/min and Bob at a speed of 72 m/min, when Bob finally catches up to Alex, it will be at point _____ (fill in "A", "B", "C" or "D").



【解析】 追及时间是： $90 \times 3 \div (72 - 63) = 270 \div 9 = 30$ (分)

乙走的路程是： $72 \times 30 = 2160$ (米)， $2160 \div 90 = 24$ (条)

也就是乙从 B 点出发，经过了 24 个 90 米，也就是走了 $24 \div 4 = 6$ (圈)

因此，当乙第一次追上甲时，正好在起点 B。

7. A boat can travel 60 km downstream and 80 km upstream in 3.2 hours. The boat then travels 80 km downstream and 40 km upstream in another 2.6 hours, the speed of the boat in still water is _____ km/h.

【解析】考虑第 2 个过程的 2 倍，顺流航行 160 千米，逆流航行 80 千米共用了 5.2 小时，则比第 1 个过程多走了 100 千米，多花了 2 小时，所以顺流速度是 50 千米/时。

逆流速度： $80 \div (3.2 - 60 \div 50) = 40$ 千米/时。

船在静水中的速度： $(50 + 40) \div 2 = 45$ 千米/时。

8. By utilizing each of the digits 2, 3, 4, 5, and 6 once, fill in the corresponding blanks to multiply a 3-digit number by a 2-digit number. When the product is the largest, the sum of the two numbers is _____.

【解析】数位不一致，末尾添 0 补齐，然后百位填 6 和 5，十位填 4 和 3，个位 2 和 0。

根据和一定差小积大，乘积最大为： 630×542 ，所以此时这个算式是： 542×63 ，和为 605。

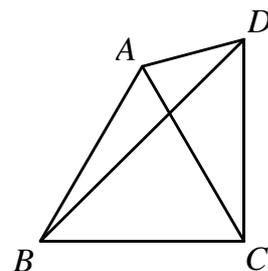
Part II. Fill in the blanks ($6' \times 8 = 48'$)

9. To complete the work, 80 automobiles had to travel through a 120-meter tunnel. If each car is 10 meters long and there must be a 20-meter space between them, it would take the convoy _____ minutes to travel through the tunnel at 500 meters per minute.

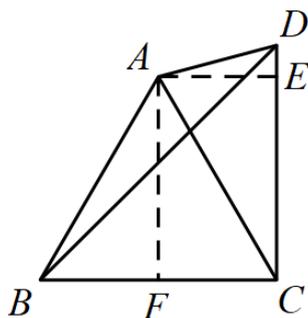
【解析】 $80 \times 10 = 800$ (米)， $20 \times (80 - 1) = 20 \times 79 = 1580$ (米)，所以车队总长为 $800 + 1580 = 2380$ (米)

需要： $(120 + 2380) \div 500 = 2500 \div 500 = 5$ (分钟)

10. The figure depicts an equilateral triangle ABC and an isosceles right triangle BCD, if BC=18, the area of the triangle ACD is _____ .



【解析】如下图：



过 A 分别作 $\triangle ACD$ 、 $\triangle ABC$ 的高 AE 、 AF ，所以 $AE = CF = \frac{1}{2}BC = \frac{1}{2} \times 18 = 9$ ，因为， $\triangle ABC$ 是等边三角形， $\triangle BCD$ 是等腰直角三角形，所以 $CD = BC = 18$ ，所以 $\triangle ACD$ 的面积：
 $18 \times 9 \div 2$
 $= 9 \times 9$
 $= 81$

11. A teacher gives out fruits to some pupils; if everyone gets 10 apples, there will be 8 left; if everyone gets 11 peaches, 4 more peaches will be needed. It is known that there are 5 more apples than peaches, there are _____ apples and peaches in total.

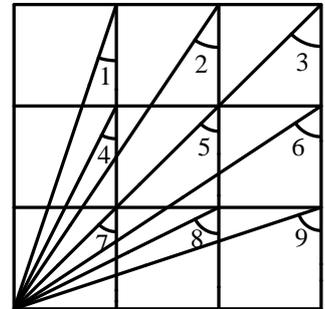
【解析】把两次都转化为分苹果，根据“桃子若每位同学分 11 个，结果就会少 4 个，已知苹果比桃子共多 5 个”转化为桃子若每位同学分 11 个，结果就会还剩下 $5 - 4 = 1$ 个，这样总差额是 $8 - 1 = 7$ 个，每份的差额是 $11 - 10 = 1$ 个，所以人数是 $7 \div 1 = 7$ 人，
 苹果个数： $10 \times 7 + 8 = 78$ （个）
 桃子个数： $11 \times 7 - 4 = 73$ （个）
 一共： $73 + 78 = 151$ （个）。

12. In the vertical algorithm below, different Kanji corresponds to different digits from 0 to 9, and the same Kanji to the same number. The 4-digit number that “方了个田” corresponds to is _____ .

$$\begin{array}{r}
 \text{个 田} \\
 \times \quad \text{田 个} \\
 \hline
 \text{田 了 田} \\
 \text{有 个 田} \\
 \hline
 \text{方 了 个 田}
 \end{array}$$

【解析】研究 $\overline{\text{个田}} \times \overline{\text{田个}} = \overline{\text{有个田}}$ ，个位都是“田”，所以“田”可能为 0，1，5，6，但是考虑到“田”出现在首位了，所以田可能为 1，5，6，而乘积为三位数，则“田”不能为 1，“田”×“个”的个位还是“田”，则“田”只能是 5，“个”为奇数。那么 $\overline{\text{个5}} \times \overline{\text{5}} = \overline{\text{5了5}}$ ，则“个”只能为 7，算式为 $75 \times 57 = 4275$ 。

13. As indicated by the figure, a 3 by 3 square grid contains a total of 9 angles, their sum is _____ digress.



【解析】

由题意得： $\angle 3 = \angle 5 = \angle 7 = 45^\circ$ ； $\angle 1 + \angle 9 = 90^\circ$ ； $\angle 2 + \angle 6 = 90^\circ$ ； $\angle 4 + \angle 8 = 90^\circ$ ；

所以：

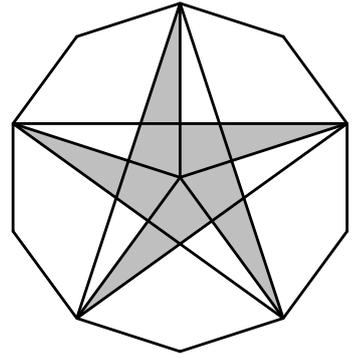
$$\begin{aligned}
 &\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 + \angle 9 \\
 &= 45^\circ \times 3 + 90^\circ \times 3 \\
 &= 135^\circ + 270^\circ \\
 &= 405^\circ
 \end{aligned}$$

14. Adam, Berry, and Conner each have one, two, or three candies. Berry says: “Adam has the most candies”, Adam responds, "I have two," and Conner responds, "I have two more candies than Berry." If one of them lied, Conner has _____ candies.

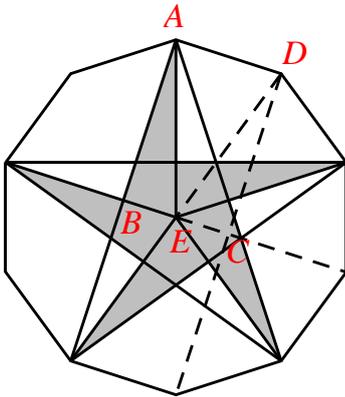
【解析】 $1 < 2 < 3$ ，这三个人中只有一个人说了谎话，根据小明和小红的表述可得，他们两个的表述只能是一真一假，不能同真，否则自相矛盾；

所以小刚的表述是正确的；那么小刚说：“我比小红多 2 根。”，只有 3 比 1 多 2，所以小刚有 3 根。

15. As depicted in the picture, the decagon's area is 2020. The shaded region's area is _____.



【解析】如图：



$$S_{\text{shaded region}} = 5S_{\triangle ABE} = 5 \times S_{\triangle AED} \div 2 = 5 \times S_{\text{total}} \div 10 \div 2 = S_{\text{total}} \div 4 = 505$$

16. Dividing the natural numbers 1 through 8 into two groups so that the difference between the sums of the two groups is 16, there are _____ possible ways.

【解析】分成的两组之和为： $(1+8) \times 8 \div 2 = 9 \times 8 \div 2 = 36$

两组的和分别为： $(36+16) \div 2 = 52 \div 2 = 26$ ； $36 - 26 = 10$

因为 $10 = 2 + 8 = 3 + 7 = 4 + 6 = 1 + 2 + 7 = 1 + 3 + 6 = 1 + 4 + 5 = 2 + 3 + 5 = 1 + 2 + 3 + 4$

相应地：

$$26 = 1 + 3 + 4 + 5 + 6 + 7 = 1 + 2 + 4 + 5 + 6 + 8$$

$$= 1 + 2 + 3 + 5 + 7 + 8 = 3 + 4 + 5 + 6 + 8$$

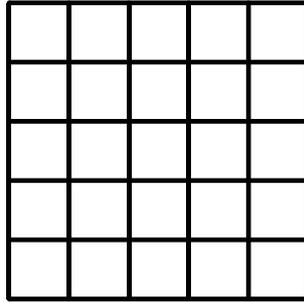
$$= 2 + 4 + 5 + 7 + 8 = 2 + 3 + 6 + 7 + 8$$

$$= 1 + 4 + 6 + 7 + 8 = 5 + 6 + 7 + 8$$

所以共有 8 种不同的分法

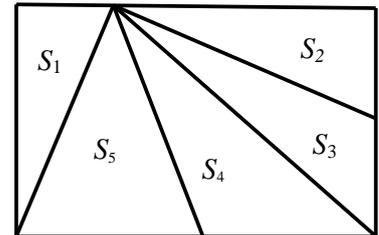
Part III. Answer questions (10'×4=40', write the necessary steps, or no points will be given)

17. As shown in the figure, 3 pieces of identical chess need to be placed in the 5 by 5 grid. If each row and column hold no more than 1 piece of chess, how many possible arrangements are there?



【解析】按照第一枚、第二枚、第三枚的次序，则有 $25 \times 16 \times 9$ 种，但是由于棋子相同，每 $3 \times 2 \times 1 = 6$ 种应当看成一种，所以总的方法数为 $25 \times 16 \times 9 \div 6 = 600$ 种

18. As shown in the figure below, a rectangle with an area of 420 is divided into 5 triangles. the triangles' area S_1, S_2, S_3, S_4, S_5 form an arithmetic sequence, find S_5 .



【解析】 $S_3 = 420 \div 5 = 84$ ， $S_4 + S_5 = 420 \div 2 = 210$ ， $S_4 = (S_3 + S_4 + S_5) \div 3 = (84 + 210) \div 3 = 98$ ，所以 $S_5 = 210 - 98 = 112$ 。

19. The natural numbers 1 through 20 are divided into two groups A and B so that the product of group A can be divided by the product of group B . What is the minimum value of $A \div B$?

【解析】 $20! = 2^{18} \times 3^8 \times 5^4 \times 7^2 \times 11 \times 13 \times 17 \times 19$ ，若使 $A \div B$ 最小，则质因数只要尽量平分即可，最小情况： $A \div B = 11 \times 13 \times 17 \times 19 = 46189$ 。

此时第 1 组：11、13、17、19、14、10、15、9、6、16、4；

第 2 组：7、20、5、18、12、3、2、8、1。

20. The following arrangement of natural numbers beginning with 5 is depicted in the figure below. The notation $distance(k)$ is defined to be the minimum number of steps needed to get from "5" to the number k , with each step required to be either horizontal or vertical. For example, $distance(19)=2$ and $distance(29)=4$, find $distance(2023)$.

...
...	21	20	19	18	17	...
...	22	9	8	7	16	...
...	23	10	5	6	15	...
...	24	11	12	13	14	...
...	25	26	27	28	29	...
...

【解析】5 是第 1 个数，所以 2023 是第 $(2023-5)+1=2019$ 个数。5 在最中间，第 1 圈右下角是第 9 个数，第 2 圈右下角是第 25 个数，第 3 圈右下角是第 49 个数，总结规律为第 n 圈右下角是第 $(2n+1)^2$ 个数。我们知道 $45^2=2025$ ， $(45-1)\div 2=22$ ，所以第 2025 个数应该在第 22 圈右下角，那么第 2019 个数应该从第 2025 个数出发，向左移动 6 格，从而 $distance(2023)=22\times 2-6=38$ 。