2022 P5 Exploration Class Selection Test

(Time: 90 min; Full marks: 120 points)

Name: _____ .   Score: _____ .

Part I. Fill in the blanks (4'×8=32')

1. $\dfrac{1}{2\times 5}+\dfrac{1}{5\times 8}+\dfrac{1}{8\times 11}+\dfrac{1}{11\times 14}+\dfrac{1}{14\times 17}=$ _____ .

【解析】原式$==\dfrac{1}{2\times 5}+\dfrac{1}{5\times 8}+\dfrac{1}{8\times 11}+\dfrac{1}{11\times 14}+\dfrac{1}{14\times 17}$。

$$=\dfrac{1}{3}\times\left(\dfrac{1}{2}-\dfrac{1}{5}+\dfrac{1}{5}-\dfrac{1}{8}+\dfrac{1}{8}-\dfrac{1}{11}+\dfrac{1}{11}-\dfrac{1}{14}+\dfrac{1}{14}-\dfrac{1}{17}\right)$$

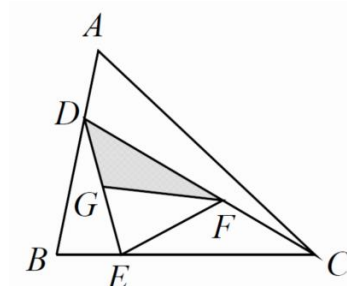$$=\dfrac{1}{3}\times\left(\dfrac{1}{2}-\dfrac{1}{17}\right)=\dfrac{5}{34}$$

2. Given a 12-digit number 314159265358, if Watson removed six digits to make it a 6-digit number, the maximum value of this 6-digit number is _____ .

【解析】高位尽量大，首位最大为 9，则第 2 位最大为 6，答案为 965358。

3. At what time between 4 and 5 o'clock will the hour hand and the minute hand of a clock point exactly to the opposite directions? Ans: _____ .

【解析】$\left(120+180\right)\div 5.5=54\dfrac{6}{11}$ （分钟）。

4. As shown in the figure, the area of the triangle ABC is 24 square centimeters, if BD=2DA, CE=3EB, DF:FC=3:2 and G is the midpoint of DE, the shaded area of the triangle DFG is _____square centimeters.
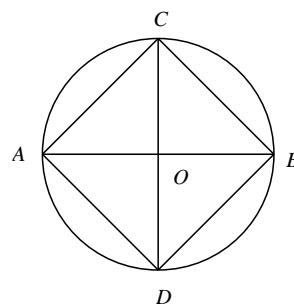

【解析】 $24 \times \frac{2}{3} \times \frac{3}{4} \times \frac{3}{5} \times \frac{1}{2} = \frac{18}{5}$ （平方厘米）。

5. A 5-digit number $\overline{8a25b}$ is both a multiple of 8 and a multiple of 9. The value of a+b is _____.

【解析】由于是 8 的倍数，所以 b 只能为 6;结合 9 的倍数，可以确认 a 为 6; 答案为 12。

6.As shown in the figure, from A to B, there are _____ different paths if a point cannot be passed more than once.
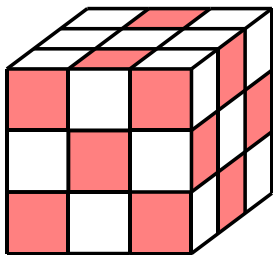

【解析】5×5=25（种）。

7. Amelia and Brandon are competing in a shooting contest; for each target they hit, they receive 20 points, while for each one they miss, they lose 12 points. With 10 shots for each of them and their combined score is 208, Amelia has a 64-point advantage over Brandon, Brandon hit the target _____ times.

【解析】乙得(208-64)÷2=72（分），若全中，则应得 200 分，实际少了 200-72=128（分），

说明脱靶 128÷(20+12)=4（次），故乙打中 10-4=6（发）。

8. Each side of a cube has been divided into 9 squares. If we use the color red, yellow and blue to color these squares such that squares sharing a same side cannot be the same color, the maximum number of square we can color in red is _____.
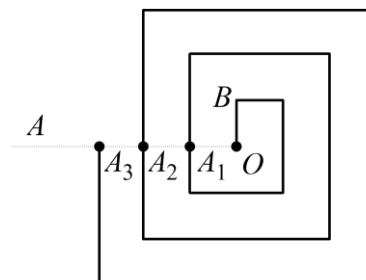
【解析】5×2+4×2+2×2=22（个）。

Part II. Fill in the blanks (6'×8=48')

9. $0.1\dot{2}\dot{3} =$ _____.

(Rewrite this decimal as a proper fraction, where the numerator and denominator are both integers.)

【解析】原式$=\dfrac{123}{999}=\dfrac{41}{333}$

10.The spiral corridor shown in the schematic diagram below has a width of 1. The length of OB is also 1 and the spiral line intersects with the ray OA at the point A_1, A_2, A_3,…. If the first spiral starts from O and ends at point A_1 (with length equal to 7), the second spiral starts from A_1 and ends at point A_2 and vice versa. The length of the tenth spiral would be _____.

【解析】每一圈比前一圈长 4×2=8，故第 10 圈的长为 7+(10-1)×8=79。

11.There are 1123 pupils sitting a selection test, Amber says: "At least 10 pupils are from the same school". To make sure that her statement is correct, there are at most _____ schools signed up for this selection test.

【解析】124×9=1116，125×9=1125，故答案为 124。

12. From 1, 2, 3, 4, 5, 6, 7 and 8, choose five numbers to complete the 1-by-5 grid. Each number can only be used once. If the number in the grey box must be greater than its two neighbors on the left and right, there are _____ possible arrangements.

【解析】 $C_8^5 \times (12+4) = 896$ （种）。

13. Dividing the 8 numbers 40, 44, 45, 63, 65, 78, 99 and 105 into two groups such that the product of two groups is the same, the sum of the group that contains 99 is _____.

【解析】40=2³×5，44=2²×11，45=3²×5，63=3²×7，65=5×13，78=2×3×13，99=3²×11，105=3×5×7，

设 99 在第 1 组，则 44 在第 2 组，则 40 在第 1 组，78 在第 2 组，则 65 在第 1 组，105 和 45 在第 2 组，则 63 在第 1 组，所以第 1 组的和为 99+40+65+63=267。

14. Of the 60 individuals, 40 play table tennis, 45 play badminton, 48 play volleyball, and 22 play all three sports. At most _____ of them do not play any of these three sports.

【解析】(40-22)+(45-22)+(48-22)=67 （人次），67=2×33+1，60-22-34=4 （人）

15. Frank has 10 identical candies, if he eats at least 1 candy everyday starting from today, there are _____ ways he can finish all of them.

【解析】我们将 10 块大白兔奶糖从左至右排成一列，

如果一天吃完，则无须分段；如果两天吃完，则需分成 2 段；

如果 3 天吃完，则需分成 3 段；……；如果 10 天吃完，则需分成 10 段；

分别需要在 9 个空隙中插入 0 个、1 个、2 个、……、9 个挡板；

最终相当于 9 个空隙中的每个空隙都可以插入，也可以不插入，故共有 2^9=512 种不同的插入方法。

16. A 3-digit number is divided by 7, 8, 9 and the remainders are written on three different paper respectively. Three intelligent and honest children named Adam, Barbara, and Caitlyn each received one of these papers, and they can only see the information that is on their paper. They gave the following statements, according to the order below.
Adam: "This 3-digit number is not a multiple of 3."
Barbara: "This 3-digit number must be an odd number."
Caitlyn: "I already know what this 3-digit number is, and I can tell you it is a composite number."
The 3-digit number is _____.

【分析】由甲的话可以推出他拿的纸上是除以 9 的余数，但具体是多少？如果拿到的数小于 8，则无法判断自己拿到的是除以 9 的余数；所以拿到的是 8.

在此基础上，乙说这个数一定是奇数，说明他拿到的纸张是除以 8 的余数，且为 7.

至此，丙知道手中拿到的数是除以 7 的余数，且这个三位数加上 1 能被 72 整除，

把 72 的倍数减去 1 后，依次为 143，215，287，359，431，503，575，647，719，791，863，935，它们除以 7 的余数依次是 3，5，0，2，4，6，1，3，5，0，2，4，

显然如果是 3，5，0，2，4 则不能确定是哪个数，所以只剩下 503 和 575，

而 503 是质数，所以这个三位数是 575.

Part III. Answer questions (10'×4=40', write the necessary steps, or no points will be given)

17. In the field of cryptography, plaintext refers to the unencrypted data and cyphertext to the encrypted data. The table below shows the encryption algorithm used to convert the English alphabet into numbers:

a	b	c	d	e	f	g	h	i	j	k	l	m
1	2	3	4	5	6	7	8	9	10	11	12	13
n	o	p	q	r	s	t	u	v	w	x	y	z
14	15	16	17	18	19	20	21	22	23	24	25	26

We are given the following formula: When $1 \le x \le 26$, if x cannot be divided by 2, then $x' = \dfrac{x+1}{2}$; if x can be divided by 2, then $x' = \dfrac{x}{2} + 13$. If an English alphabet corresponds to a number x, and by using the formula above, we can turn this number x into x'. For example, if the plaintext is a single alphabet *"g"*, then $g \to 7 \to \dfrac{7+1}{2} = 4 \to d$, and therefore the cyphertext of *"g"* is *"d"*.

By using the formula above, what is the cyphertext that corresponds to the plaintext *"good"*? If the cyphertext is known as *"gawqj"*, what is the original plaintext?

【解析】 $g \to 7 \to \dfrac{7+1}{2} = 4 \to d$, $o \to 15 \to \dfrac{15+1}{2} \to 8 \to h$, $d \to 4 \to \dfrac{4}{2} + 13 = 15 \to o$

故 good 译成的密文是 dhho。

$g \to 7 \to 2 \times 7 - 1 = 13 \to m$, $a \to 1 \to 2 \times 1 - 1 = 1 \to a$, $w \to 23 \to 2 \times (23 - 13) = 20 \to t$,

$q \to 17 \to 2 \times (17 - 13) = 8 \to h$, $j \to 10 = 2 \times 10 - 1 = 19 \to s$

故 gawqj 所对应的明文是 maths。

18. It takes worker A 12 days to finish a task alone. It takes worker B 18 days to finish the same task alone. It takes worker C 24 days to finish the same task alone. If worker A work on the task for a few days before worker B takes over. Worker B then works three times as long as the worker A did before worker C takes over. Worker C then works two times as long as worker B did and the task is finally completed, how many days did it take?

【解析】设甲工作 x 天，则乙工作 3x 天，丙工作 6x 天，总共用了 10x 天

由题意知 $\dfrac{1}{12} \times x + \dfrac{1}{18} \times 3x + \dfrac{1}{24} \times 6x = 1$ ，解得 x=2，总共用了 20 天。

19. A store bought some toys at the cost of $36 each. If the store sells a toy for $80, it will sell 50 toys. If every $2 reduction in price results in the sale of an additional 5 toys, what should be the selling price in order to maximize profit?

【解析】设下降 x 个 2 元，则会多卖出 x 个 5 件，总利润为 $(80-2x-36) \times (50+5x) = 10(22-x)(10+x)$ 元，

当且仅当 22-x=10+x，即 x=6 时，总利润最大，故应将价格定为 80-2×6=68（元/件）

20. It is given that the length of the edges of a big cuboid are all natural numbers, if we color all six sides of it in red and cut it into small cubes with edge length 1, there are 12 small cubes with none of its sides red, 32 small cubes with two sides in red, how many small cubes are there with only one side in red?

【解析】设"内核"的规格为 a×b×c，则 $\begin{cases} abc = 12 \\ 4(a+b+c) = 32 \end{cases}$，所以"内核"只能是 1×3×4，

一个面涂红色的有 (1×3+3×4+1×4)×2=38（块）