

APMOPS 2015 Round 1

Time Duration: 2 hours

Name: _____

Marks: _____

1. Albert and Ben start running on a 200-metre circular track at the same point and at the same time, in a clockwise direction.

The speed of Albert and Ben are 6 m/s and 4 m/s respectively.

Determine the number of times Albert overtake Ben in the first 16 minutes after the start of run.

2. A cuboid has its length, breadth and height in the ratio of 4:3:2 .

If the sum of all the edges is 72 cm, what is the volume of the cuboid?

3. Evaluate $(101 + 234 + 567) \times (234 + 567 + 89) - (101 + 234 + 567 + 89) \times (234 + 567)$.

4. There are red, yellow and green balls in a container.

The red balls are all marked with the number '4'.

The yellow balls are all marked with the number '5'.

The green balls are all marked with the number '6'.

If 8 balls are drawn and the sum of the numbers on the balls is 39, how many of these are red colour balls?

5. Given $\frac{5}{9} < \frac{9}{A} < 1$, where A is an integer.

Find the number of possible values of A.

6. A given month has 31 days.
The number of Mondays and Fridays are the same.
Which day of the week is the 10th of the month?

7. A whole number N multiplied by 411 gives a result with last 4 digits 2015:

$$N \times 411 = \dots\dots 2015.$$

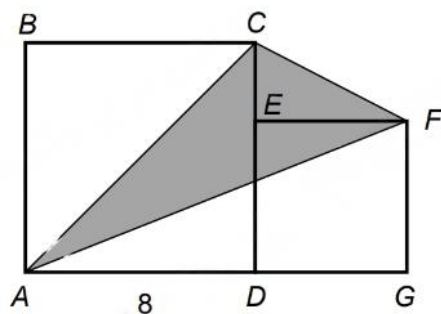
Find the minimum value of N .

8. A standard soccer ball is a sphere that is made up of a total of 32 regular pentagons and hexagons. The diagram below shows a sample of a standard soccer ball where the polygons shaped in black are pentagons and the rest hexagons.

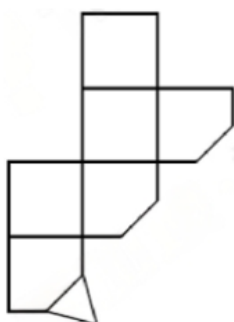
What is the number of pentagons on the surface of this soccer ball?



9. The diagram below shows two squares $ABCD$ and $DEFG$ joined at the side ED . The length of AD is 8 cm. Find the area of triangle ACF .



10. During a class gathering, the class monitor, Lizz, bought some bottle drinks. Lizz bought the bottle drinks from a convenient store during a special offer. For 5 empty bottles, Lizz can exchange 1 free bottle drink from the store. Some of the bottle drinks are from the exchange of empty bottles. If the class drank 109 bottles, what is the minimum number of bottles Lizz bought?
11. The figure shows the net of a polyhedron. How many edges does this polyhedron have?

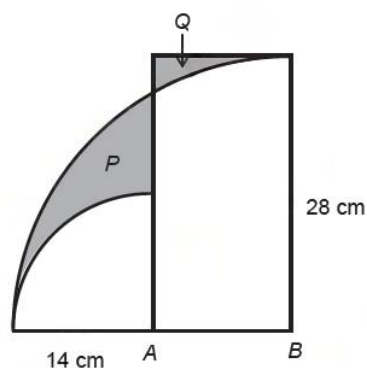


12. Give the number 2^{29} is formed by 9 different digits.
Which digit is not in this number?
13. In a class of 52 students, there are 30 swimmers, 35 cyclists and 42 playing table tennis.
Each student participates in at least two sports.
How many students play all three sports?
14. A transport company has two types of delivery trucks: a large truck that can carry 7 tons of goods and a small truck that can carry 4 tons of goods.

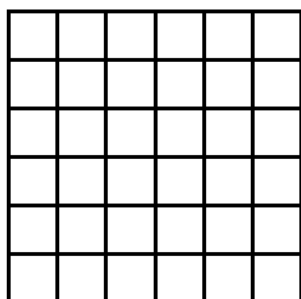
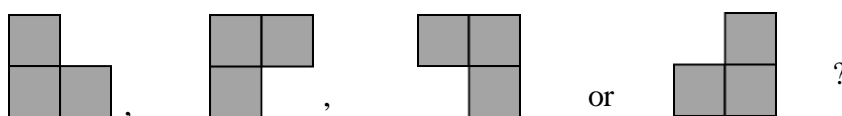
The fuel consumption for a large truck is 14 litres per trip while that for a small truck is 9 litres per trip.

What is the minimum total fuel consumption needed for the company to deliver 89 tons of goods?
15. Excluding an interior angle, the sum of the remaining interior angles of a convex polygon is 2015° .
What is the angle of the excluded interior angle?

16. In the figure below, the points A and B are the centres of two quadrants of radius 14 cm and 28 cm respectively. Find the difference of the areas of the shaded regions P and Q . (Take $\pi = \frac{22}{7}$)

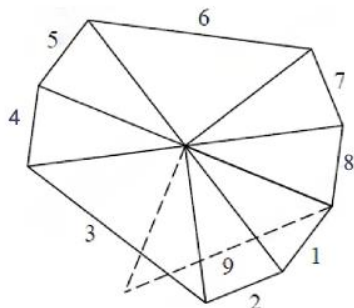


17. In a 6×6 grid, how many ways can we place a single shape of

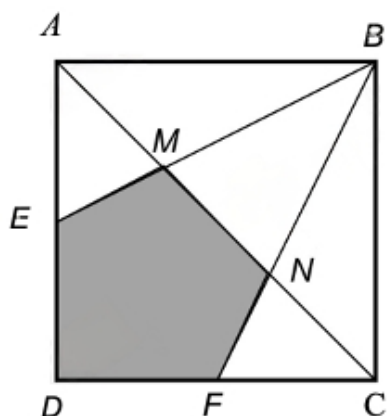


18. When dividing two whole numbers, the quotient is 15 and the remainder is 5. The sum of the dividend, divisor, quotient and remainder is 2169. Find the value of the divisor.

19. The figure below is formed using two types of isosceles triangles.
 Triangles 1 and 2 are 30° isosceles triangles, triangle 3 is right-angled isosceles.
 Triangles 4 and 5 are 30° isosceles triangles, triangle 6 is right-angled isosceles.
 And the triangles continue in this pattern.
 If the n^{th} triangle will coincide with triangle 1, what is the smallest possible value of n ?

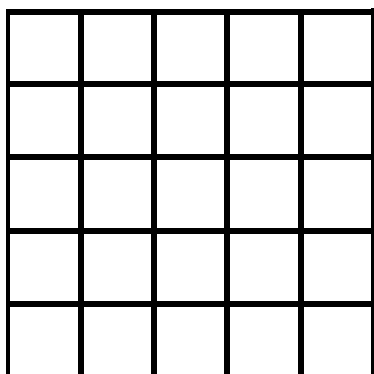


20. In the figure below, $ABCD$ is a square; E and F are midpoints of AD and DC respectively.
 The lines BE and BF cut diagonal AC at M and N respectively.
 If the area of the square is 48 cm^2 , find the area of the shaded region.



21. Four soccer teams played in a tournament where they played against each other only once.
In each match, the winning team was awarded 3 points while the losing team had 0 points.
If the two teams played to a draw, each team received 1 point.
After all the matches, the points of the four teams are consecutive numbers. Find the sum of these four numbers.
22. A huge water tank was fully filled with water and it has 5 taps to drain the water out.
If the 1st to 4th taps are all turn on, it takes 6 hours to empty the tank.
If the 2nd to 5th taps are all turn on, it will take 8 hours to empty the tank.
If only the 1st and 5th taps are turn on, it will take 12 hours to empty the tank. If only the 5th tap is turn on, how many hours will it take to empty the tank?
23. The product $5 \times 10 \times 15 \times 20 \times \dots \times 2010 \times 2015$ has how many consecutive zeros at the right when fully evaluated?

24. In the 5×5 grid below, some squares are painted such that any 3×3 grid in it has 4 squares painted.
 What will be the least number of squares painted in the grid?



25. There are 49 balls each labelled with numbers 1 to 49.
 Some balls are selected to form a circle where the product of any two adjacent balls is less than 100.
 What is the maximum number of balls that can be chosen?
26. Three towns A, B and C lie on a straight road.
 Peter and Charles started driving from town A and B at the same time, respectively, to town C.
 The ratio of the speed of Peter to Charles is $3:2$.
 Peter and Charles reached town C at 9 am and 7 pm respectively, on the same day.
 What was the time when Peter overtook Charles?

27. The alphabets a , b and c represent digits from 0 to 9.
 They can be identical digits.
 The recurring decimal $0.\overline{abc}$ can be converted into a fraction.
 Find the number of different numerators.
28. Five cards A , K , Q , J and 10 was placed into five envelopes labelled A , K , Q , J and 10.
 If each envelope contains only 1 card, how many ways are there for the last card to be placed in the wrong envelope?
29. How many 3-digit numbers are divisible by 3 and each contains at least one digit '3'?
30. In the figure below, $BDEC$ is a straight line, $\angle BAD = \angle DAE = 12^\circ$ and AC is perpendicular to AD .
 If $BC = AB + AE$, find $\angle ABC$.

