**NMOS 2015 Special Round**

Time Duration: 1.5 hour   Name: _____   Marks: _____

1. Benjamin had 30% more beads than Chloe and 50% fewer beads than Annie. After Annie and Benjamin gave 90 beads and 95 beads respectively to Chloe, Benjamin had $\frac{1}{3}$ as many beads as Annie. How many beads did Chloe have at first?

2. In the triangle ABC given below, D, E and F are the mid points of AB, AC and AE respectively. Given that BG : GC = 1 : 2 and that the area of triangle CDF is 126 cm², find the area, in cm², of the quadrilateral CGDE.
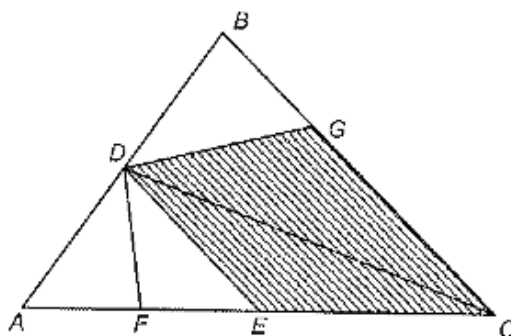

3. Alex, Brandon and Cayden are to complete a 940-component LEGO set, where the time spent on each component is the same. If each of them completes the LEGO set individually, the ratio of the time they spent is 3:4:5 respectively. Now they are to work together to complete the LEGO set. How many components will Alex form?

4. It is known that u is a whole number not more than 100 and the average of $u, 101, 102, 103, \ldots, 119$ and 120 is a whole number. Find the sum of all possible values of u.

5. Note that $101^2 = 10201$. This is an example of a 3-digit whole number having its last 2 digits unchanged when it is squared.

   How many 3-digit whole numbers from 100 to 999, including 101, have their last 2 digits unchanged when they are squared?

6.
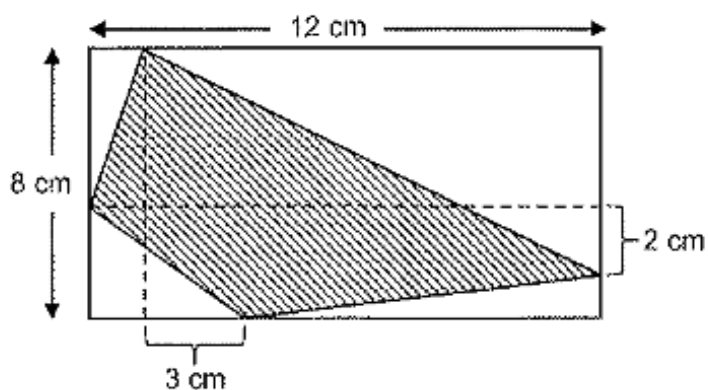
   The diagram above shows a rectangle of dimension 8 cm $\times$ 12 cm. Find the area, in cm^2, of the shaded region.

7. On Sunday morning, John decided to visit the national museum, but he was not aware of the actual time. John set a newly purchased alarm clock as 8:00 and left home. Upon arriving at the national museum, John was told by the security guard that the actual time was 9:15. John stayed in the museum for one hour and returned home. He noticed that his new alarm clock showed 10:50 when he arrived home. Suppose John's journey from his home to the museum took the same amount of time as his journey from the museum back to his home. What is the actual time when he left his home? (For example, if you think he left home at 7:30, write your answer as 730).

8. A palindromic number is a number that stays unchanged whether it is read from left to right or from right to left. For example, 121 is such a palindromic number. How many 3-digit palindromic numbers are not divisible by 11?

9. There are 2 inlets (namely A and B) in a pool. If only inlet A is open, it takes 36 minutes to fill up the empty pool. If only inlet B is open, it takes 48 minutes to fill up the empty pool. Now, inlets A and B will be open in terns, according to such and order, open inlet A for 1 minute, B for 2 minutes, A for 2 minutes, B for 1 minute, A for 1 minute, B for 2 minutes… so on and so forth. How long, in the nearest minutes, does it take to fill up the empty pool?

10. Positive odd numbers are arranged in the following manner:

1st row:				1			
2nd row:			3	5	7		
3rd row:		9	11	13	15	17	
4th row:	19	21	23	25	27	29	31

$\vdots$

Which Row does 2015 appear in?

11. In a specific math Olympiad training class, the proportion of female students is more than $\frac{4}{15}$, but less than $\frac{3}{10}$. What is the smallest possible number of pupils in the class?

12. Each of the numbers 1, 2, 2, 4, 4, 8, 8 is placed in a single region of the diagram below such that the product of the numbers in each circle is the same. The example below gives the common product $1 \times 2 \times 4 \times 8 = 64$.
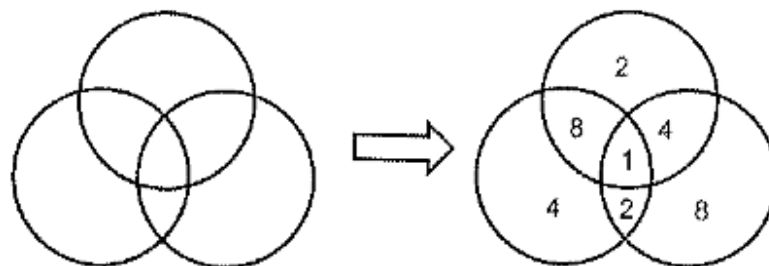

Let the largest product be X and the smallest product be Y. Find the value of $X - Y$.

13. Each of the letters E, I, N, S, T, V, W, X and Y represents a different one of the digits 0, 1, 2, 3, 4, 5, 6, 7 and 8 such that

$$
\begin{array}{r}
S\ I\ X \\
S\ E\ V\ E\ N \\
+\ \ \ \ S\ E\ V\ E\ N \\
\hline
T\ W\ E\ N\ T\ Y
\end{array}
$$

Given that $S = 6$ and $E = 8$, find the 4-digit number "$TIVY$".

14. In the parallelogram $ABCD$ given below, AB is parallel to CD while AD is parallel to BC. It is given that $\frac{CE}{AD} = \frac{CF}{AF} = \frac{EF}{DF} = \frac{1}{2}$. Given that the area of $ABEF$ is 25 cm^2, find the area of the parallelogram $ABCD$.
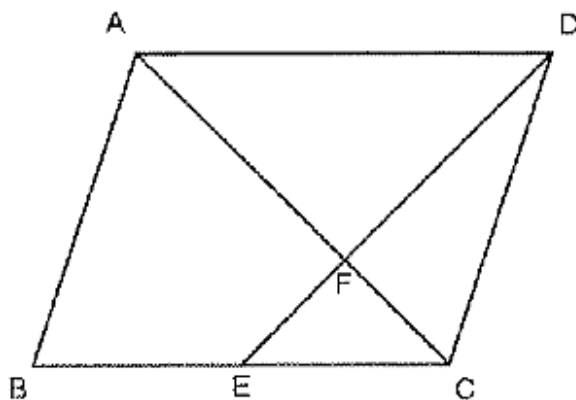
15. Study the following pattern:

$$1 = 1 = 1^2$$
$$1+3 = 4 = 2^2$$
$$1+3+5 = 9 = 3^2$$
$$1+3+5+7 = 16 = 4^2$$
$$1+3+5+7+9 = 25 = 5^2$$

Given that $23 + 25 + 27 + \cdots + (2k - 1) = m^2$, where k and m are whole numbers, $k > 30$, find the value of m.

16. There are exactly 8 factors of 24, i.e. 1, 2, 3, 4, 6, 8, 12 and 24. How many whole numbers from 1 to 100, including 24, also have exactly 8 factors?

17. It is known that u and v are whole numbers and that $2u + 3v$ is not more than 100. Find the largest value of $u \times v$.
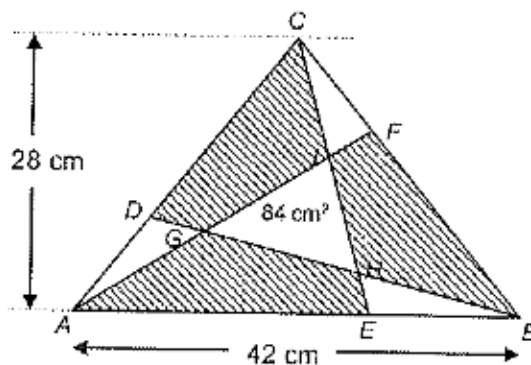
18. In the following equation, each letter represents a distinct digit

$$5\times \overline{ABCDEF} = 6 \times \overline{EFABCD}$$

Given that $B = 2$ and $D = 0$, find the 4-digit number "$FACE$".

19. In the diagram below, the base $AB = 42$ cm and the height of the triangle is 28 cm. It is known that the area of the triangle GHI is 84 cm^2.
Given also that $\frac{FC}{BC} = \frac{BE}{BA} = \frac{AD}{AC} = \frac{1}{3}$, find the total area of the shaded regions.

20. Four soccer teams, A, B, C and D, played a round-robin tournament, i.e. every two team played exactly one match. 3 points are awarded to a team winning a match, and 0 points to the losing team. If the game is drawn, each team receives 1 point.

Suppose A, B, C and D scored a, b, c and d points respectively, where a > b > c > d. Find the largest possible value of a × b × c × d.