**SMOPS 2014 Round 1**
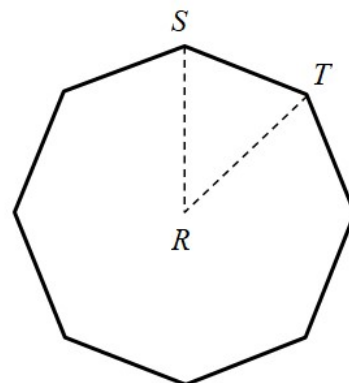
Time: 2 hours   Full marks: 30×5′=150′   Name: _____   Marks: _____

1.  The figure shows a regular octagon.
    Find the value of angle *SRT* in degrees, where *R* is the centre of the octagon.



2.  Which of the following statement is correct?

    (1) $\dfrac{15}{19} < \dfrac{7}{9} < \dfrac{3}{4}$

    (2) $\dfrac{3}{4} < \dfrac{15}{19} < \dfrac{7}{9}$

    (3) $\dfrac{7}{9} < \dfrac{15}{19} < \dfrac{3}{4}$

    (4) $\dfrac{3}{4} < \dfrac{7}{9} < \dfrac{15}{19}$

    (5) $\dfrac{7}{9} < \dfrac{3}{4} < \dfrac{15}{19}$

3.  Find the smallest positive integer with sum of digits equal to 29.

4. Abel and Jim run a race together on a 300-metre circular track.They start simultaneously at the same point.
   Abel and Jim run at constant speeds of 5 m/s and 4.2 m/s respectively.
   How many full laps has Abel run before he is able to overtake Jim for the firsttime?

5. Only one of the following four numbers is a perfect square.
   Which one is it?
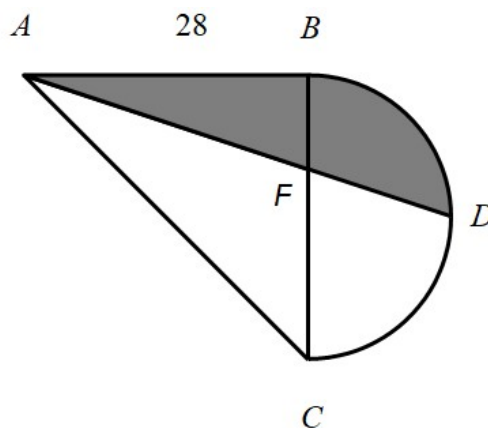   (1) 76186          (2) 750235          (3) 921438          (4) 2660161

6. In the diagram, ABC is a right-angled isosceles triangle with AB=28 cm. BC is the diameter of the semi-circle and point D is the midpoint of arc BC. Find the area of the shaded region in cm$^2$.
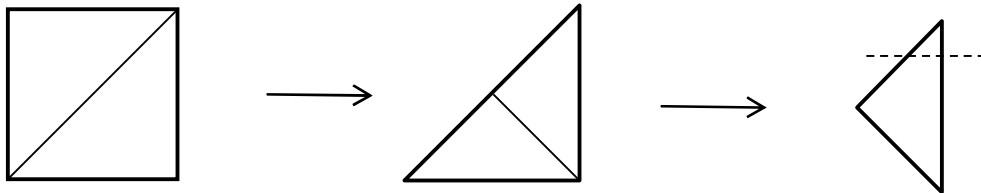
   (Take $\pi=\dfrac{22}{7}$)



7. Three chess pieces, each of colour red, black and white, are to be placed on a 7×7 chess board.
   If any two of the three pieces cannot be placed in the same row or the same column, how many ways are there to place the three chess pieces?
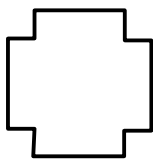
8. There are six bags containing 18, 19, 21, 23, 25 and 34 balls respectively.
   One bag contains only red balls while the other five bags contain only blue balls.Jason takes three bags and Jamie takes two bags.
   The remaining bag contains only red balls.
   It is known that Jason has twice as many blue balls as Jamie does. Find the number of red balls in the remaining bag.

9. A water tank can be filled by using tap A for 8 hours followed by tap B for 15 hours. The same tank can also be filled by using tap A for 5 hours followed by tap B for 24 hours.
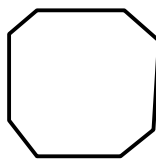   How long will it take (in hours) to fill the tank by using tap A only?

10. A square piece of paper is folded along the diagonal twice, as shown in the diagrams below.  Then a cut is made along the dotted line.
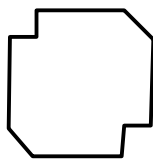
   When the paper is unfolded, which one of the following diagrams shows how
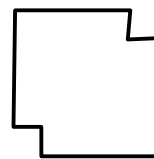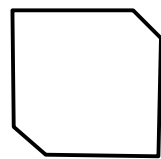
   the paper appears?

   (1)          (2)          (3)          (4)          (5)

11. Find the value of 20142013×20132014 – 20132013×20142014.

12. Alan, Ben, Chris and Daniel took part in a Science Quiz.
The quiz consisted of 5 True or False questions.
The table below shows the answers given by each of them and their scores.
Each correct answer was given 1 point.

| | Question 1 | Question 2 | Question 3 | Question 4 | Question 5 | Score |
|---|---|---|---|---|---|---|
| Alan | T | F | F | F | F | 2 |
| Ben | F | T | F | F | T | 3 |
| Chris | F | T | F | T | T | 4 |
| Daniel | F | T | T | F | T | ? |

How many points did Daniel score?

13. A computer program generated all five letter code words that can be formed by using alphabets A, B, C, D, E, F, G, X, Y, Z.
When all these words are sorted according to the alphabetical order, the following list is obtained:

AAAAA, AAAAB, …, AAAAZ, AAABA, AAABB, … ZZZZY, ZZZZZ.

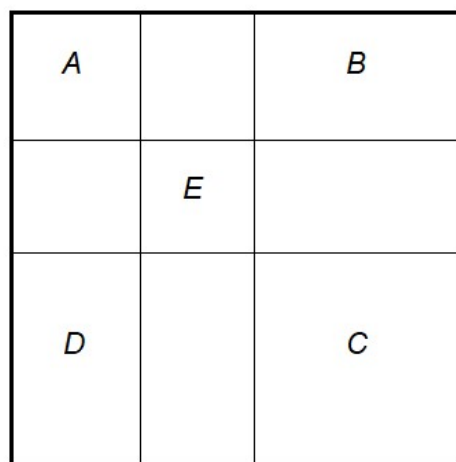Find the number of code words between CZYGB and XEFDA, not including these two code words.

14. Find the largest 3-digit whole number $N$ such that when $N$ is divided by 3, 7 and 11, the remainders are 1, 3 and 8 respectively.

15. Find the value of $10 \times (\dfrac{1}{1 \times 2} + \dfrac{5}{2 \times 3} + \dfrac{11}{3 \times 4} + ... + \dfrac{89}{9 \times 10})$

16. As shown in the figure below, a square is divided into nine regions.
Region *E* is a square and the other regions are rectangles.
If the areas of rectangles *A, B, C* are 18 cm², 63 cm² and 189 cm² respectively, find the perimeter of rectangle *D*.
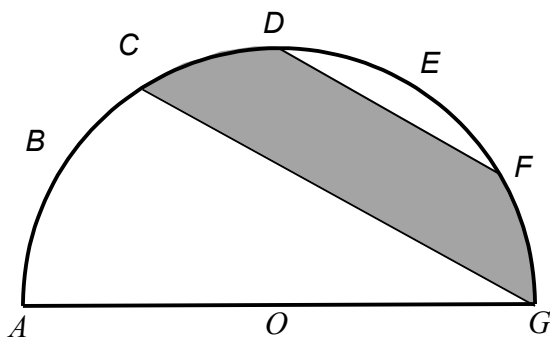


17. The sum of 47 distinct positive integers is 2014. If there are *n* even numbers among the 47 integers, what is the least possible value of *n*?

18. A 9-digit whole number $\overline{abcdefghi}$ is formed using digits 1, 2, 3, 4, 5, 6, 7, 8, 9 without repetition.

If $A = \overline{abc} + \overline{bcd} + \overline{cde} + \overline{def} + \overline{efg} + \overline{fgh} + \overline{ghi}$, find the largest possible value of *A*.
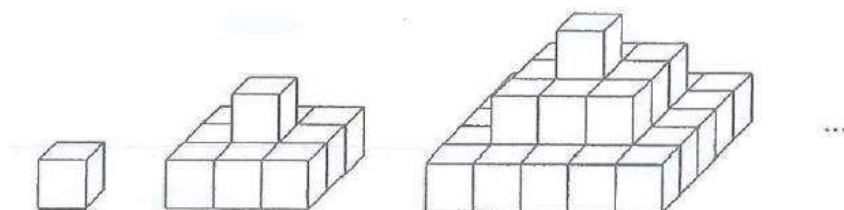
19. A semi-circle with diameter *AG* is shown in the diagram below. The entire arc of the semi-circle is divided into six equal parts by points *B, C, D, E* and *F*. *DF* and *CG* are straight lines.

   Given that the area of the semi-circle is 60 cm², find area of the shaded region in cm²
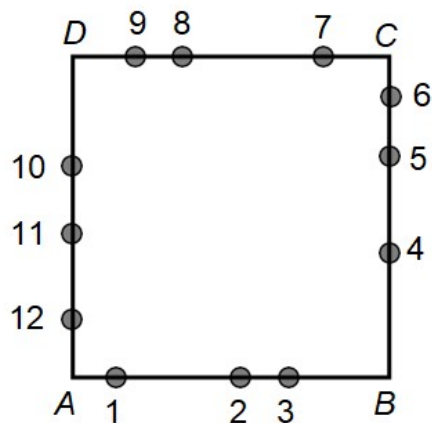


20. A shown in the diagram below, cubes with side length of 1 cm are placed together to form a sequence.

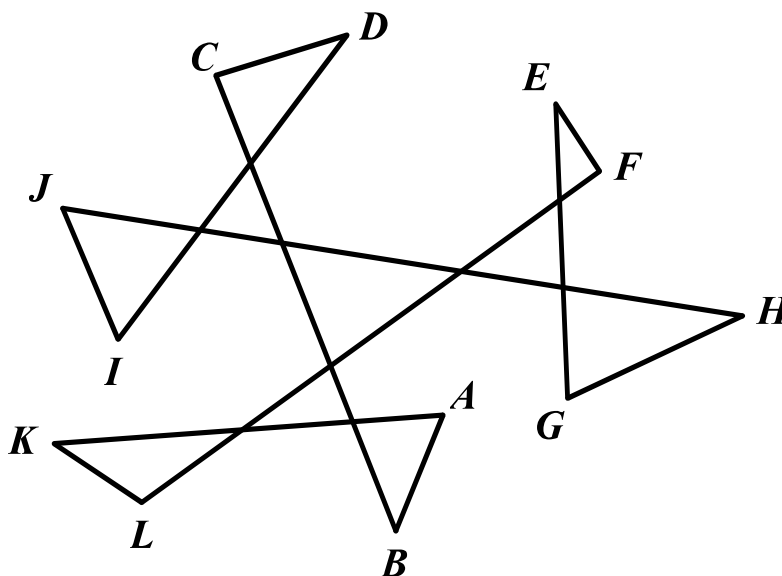   Find the surface area of the 20$^{th}$ solid in cm².



21. In the diagram, each side of the square *ABCD* is divided into four segments by the points numbered from 1 to 12.
   How many different triangles can be formed whose vertices can be any three points among the points 1 to 12?

22. In a certain calendar year, there are more Mondays than Fridays, and more Sundays than Wednesdays.
    Which day of the week is 1st March in that year?

    (1) Monday      (2) Tuesday          (3) Wednesday          (4) Thursday

    (5) Friday      (6) Saturday         (7) Sunday

23. There are 60 grams of 5% saline solution A, 60 grams of 8% saline solution B and 47 grams of 9% saline solution C. These three types of solution are mixed together to produce 100 grams of 7% saline solution.

    Find the sum of the maximum and minimum grams of solution A that can be used.

24. When a whole number is divided by 5, the remainder is $a$ ($a \neq 0$).
    When the same number is divided by 6, the quotient is $b$ and the remainder is $c$
    ($c$ can be 0.)
    If the sum of $a$ and $b$ is 11, find the sum of all numbers that satisfy this requirement.

25. Dates can be written as an 8-digit number in the form *yyyymmdd*. For example, 20140125 stands for January 25th 2014.
    How many days are there in the year 2014 such that its 8-digit representation contains equal numbers of digit 0, 1 and 2?

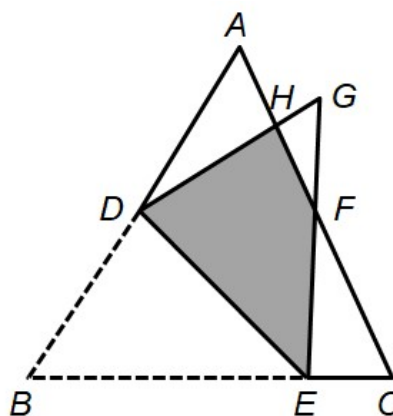26. Find the sum of all 12 angles labelled A to L in the diagram below.



27. Appending a positive integer $N$ at the end of any positive integer to form a new number (for example, appending 21 at the end of 35 gives 3521), if this new number is always divisible by $N$, then $N$ is called a 'magic number'.

Find the total number of 'magic numbers' less than 600.

28. As shown in the diagram, part of a triangle $ABC$ is folded along the line $DE$, resulting in a heptagon $ADECFGH$.

If the ratio of the area of this heptagon to the area of triangle $ABC$ is 5 : 7, and the area of the shaded region is 8 cm$^2$, find the area of triangle $ABC$.

29. From 2014 to 6999, how many integers have its sum of digits divisible by 5?

30. A bus and a truck started travelling towards each other at the same instant, from cities A and B respectively.

When they met along the road after 6 hours, the bus was 240 km away from city B.

Upon reaching city B, the bus stopped for an hour for maintenance, before going back to city A.

The truck took 15 hours to travel from city B to city A, and also stopped for an hour for maintenance before going back to city B.

The bus and the truck met again on their way back to their cities of origin.

How many hours had passed between their first and second encounters?