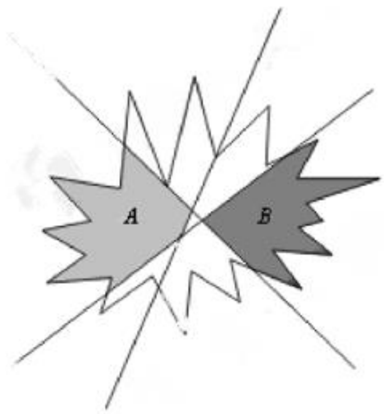
APMOPS 2016 Round 1

Time Duration: 2 hours Name: __________ Marks: __________

1. Find the value of

$$2016 + 2015 + 2014 - 2013 - 2012 - 2011 + 2010 + 2009 + 2008 - 2007 - 2006 - 2005 + \cdots + 6 + 5 + 4 - 3 - 2 - 1.$$

2. In the figure below, each line divides the figure into two equal parts.



Compare the size of the shaded regions A and B.

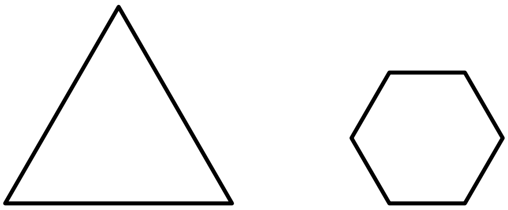
(1) $A > B$ (2) $A < B$ (3) $A = B$ (4) Insufficient information to decide

3. In this series of fractions: $\frac{1}{6}, \frac{2}{7}, \frac{3}{8}, \ldots, \frac{2010}{2015}, \frac{2011}{2016}$, how many of them are in the simplest form?

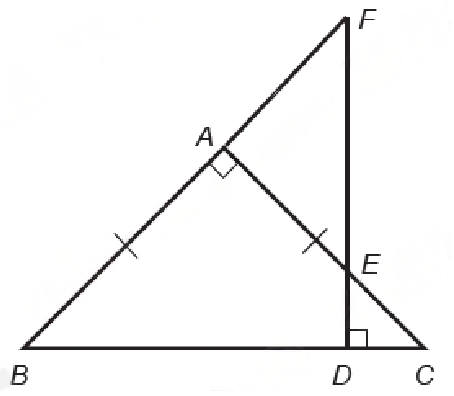
4. A new operation $\oplus$ is defined as $a \oplus b = \frac{2}{a^2} + \frac{1}{b}$. Which of the equations below is/are correct?

   (1) $2 \oplus 4 = 4 \oplus 2$
   (2) $3 \oplus 6 = 6 \oplus 3$
   (3) $4 \oplus 8 = 8 \oplus 4$
   (4) $1008 \oplus 2016 = 2016 \oplus 1008$

5. The figure below shows an equilateral triangle of sides 3 cm and a regular hexagon of sides 1 cm.
   If the ratio of the area of the triangle to the hexagon is $a:2$, find the value of a.



6. In the figure below, ABC is a right-angled isosceles triangle, where BC = 6 cm. D is the foot of the perpendicular from F to BC, intersecting the side AC at E. The extension of BA meets the perpendicular at F.
   Find the total length of $DE + DF$, in cm.

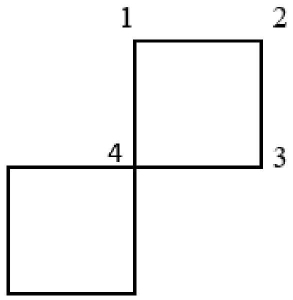


7. Mr Lim drives his car in the following way:

   drive in a straight line 20 m and turned left;
   drive in a straight line another 20 m and turned right;
   drive in a straight line another 20 m and turned right again;
   drive in a straight line another 20 m and turned right;

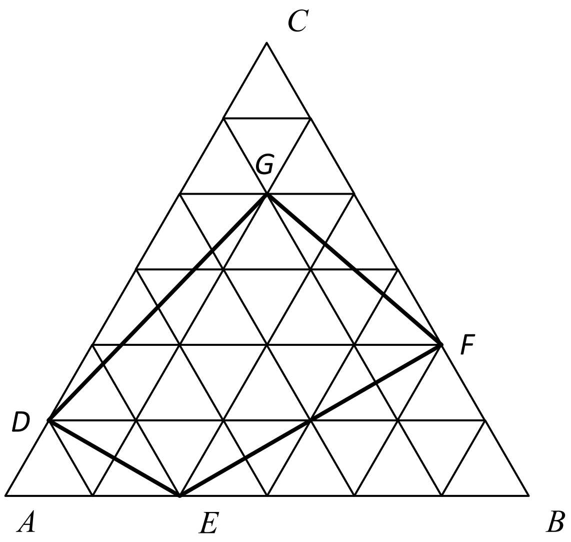
   and he repeats the process above.

   The figure below shows the route he drove. From which point did he start driving,1, 2, 3 or 4?



8. We have three cubes of lengths 3 cm, 4 cm and 5 cm and we painted all the sides of each cube red.
   If we cut these three cubes into 1-cm cubes completely, how many 1-cm cubes have at least one side painted red?

9. In the figure below, each side of the equilateral triangle *ABC* is divided into six equal parts. If the area of a unit triangle is 1 cm^2 , find the area of the quadrilateral *DEFG*, in cm^2 .



10. Albert and Ben started driving at the same time from town A and B respectively, at a constant speed towards each other.

The speed of Ben was 1.5 times that of Albert.
They met each other 12 km from the midpoint of AB.

Find AB, the distance between town A and B.

11. The areas of three different faces of a rectangular prism are in the ratio 2 : 3 : 5, and the total length of all the edges is equal to 124 cm.
Find the volume of this rectangular prism, in cm^3.

12. We wrote the first 300 natural numbers from left to right: 1, 2, 3, …, 299, 300.
We then removed all the numbers that are divisible by 5 or 7.
Of the numbers left, what is the 123rd number?

13. In a triangle *ABC*, 2016 interior points are placed, such that in the 2019 points (including *A*, *B*, C), no three points lie on a straight line.
Divide triangle *ABC* into smaller triangles using these 2019 points as vertices of the triangles.
What is the greatest number of triangles that we can get?

14. Find the value of:

$$84 \times \left(\frac{1}{1 \times 3} - \frac{2}{3 \times 5} + \frac{3}{5 \times 7} - \frac{4}{7 \times 9} + \cdots + \frac{9}{17 \times 19} - \frac{10}{19 \times 21}\right)$$

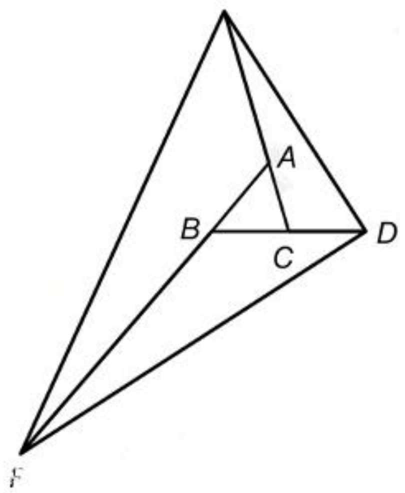
15. Given three primes numbers a, *b*, c (can be of the same values) where the product *abc* is the sum of 13 consecutive natural numbers.

Find the smallest possible value of $a + b + c$.

16. How many consecutive zeros are there in the product:

$$1 \times 4 \times 7 \times 10 \times \ldots \times 697 \times 700 \text{ ?}$$

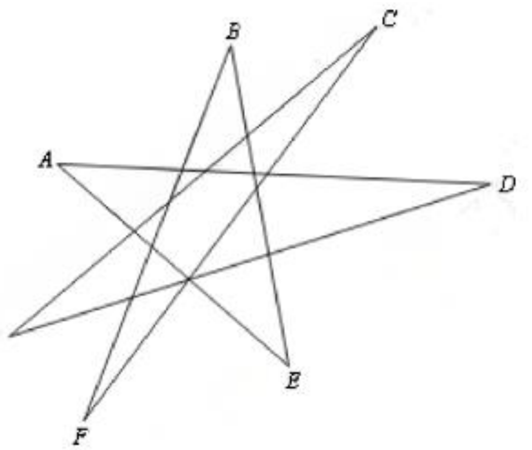
17. In the figure below, the area of triangle ABC is 1 cm^2. Extend BC, CA, AB to points D, E, F such that $BD = 2BC$, $CE = 3CA$, $AF = 4AB$.
Find the area of triangle DEF, in cm^2.



18. Stacking 6 one-cent coins, it will have the same height as 5 two-cent coins. Stacking 4 one-cent coins, it will have the same height as 3 five-cent coins. Using one-cent, two-cent and five-cent coins, Sam places only coins of the same value into three stacks of equal heights.
If he used a total of 124 coins, what is the total value of all these coins in cents?

19. How many 10-digit whole numbers are there, where in each number, the product of the 10 digits is 2^{27}?

20. In the figure below, find $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G$.



21. Three soccer teams A, B and C played in a tournament where they played against each other only once.
After all the matches, a sports reporter gathered the following information: Team A scored a total of 6 goals and let in fewer than 5 goals;
Team B let in a total of 3 goals; Team C scored at least 7 goals.
What was the result when Team A played against Team C - (:   ) ?

22. A positive integer N can be divided by 18 numbers from the first 20 natural numbers (1 to 20).
The two numbers that cannot divide N happened to be consecutive numbers.Find the sum of these two numbers.

23. Mr Dan lives in town A and he wants to travel to town F which is 60 km away. He plans to stop along the way at towns B, C, D and E, that are 4 km, 16 km, 33 km and 44 km away respectively, from town A.

Two types of buses travel from town A to F: the mini-bus and the large coaches. The mini-bus has a stop at every 2 km and charges $3 for every 2 km travelled.The large coach has a stop at every 3 km and charges $4 for every 3 km travelled.

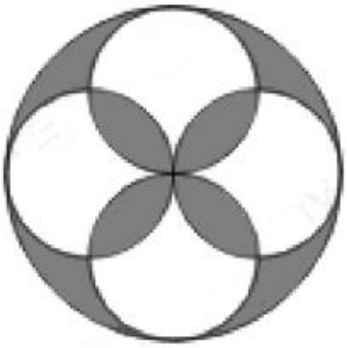
If Mr Dan used the bus service, what is the least amount he needs to pay to reach town F, visiting towns B, C, D and E along the way?

24. Find the smallest multiple of 35 that ends with '35' and the sum of all its digits is 35.

25. In the figure below, the radius of the largest circle is 7 cm.

Find the area of the shaded region, in cm^2.

(Take $\pi = \frac{22}{7}$)



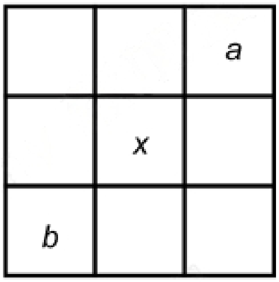
26. A computer program lists all the possible 6-letter codes using *A*, *P*, *M*, *O*, *P*, *S* in the same manner as a usual dictionary would, in alphabetical order:

AMOPPS, AMOPSP, AMOSPP, AMPOPS, ..., SPPMOA, SPPOAM, SPPOMA.

If the code POAMSP is the nth code, find the value of n.

27. The 3 by 3 grid below is filled up using all the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, exactly once, such that:
Every two consecutive digits must be at adjacent squares.
(1 is adjacent to 2, 2 is adjacent to 3, ..., 8 is adjacent to 9.)
Two squares are adjacent when they share a common side.

In the grid below, a + b = 12, find all the possible values of x.



28. In a charity show, the price of an adult ticket is \$23, a student ticket costs \$15 and a ticket for children costs \$7.
A total of \$2016 is collected from the sale of 136 tickets.
Among the tickets sold, children tickets are the most and adult tickets are the least.
How many student tickets were sold?

29. There exists five consecutive 2-digit numbers such that:
the sum of three of the numbers is divisible by 37; and
the sum of another three of the numbers is divisible by 60.
What is the largest number among these five 2-digit numbers?

30. In the figure below, how many rectangles (including squares) are there?