2023 Spring Cup
Mathematical Olympiad
PRELIMINARY ROUND

Date: 28 January 2023

Time Given: 1 hour 30 minutes

Level: Secondary 1

Name: _____

Instruction to Candidates

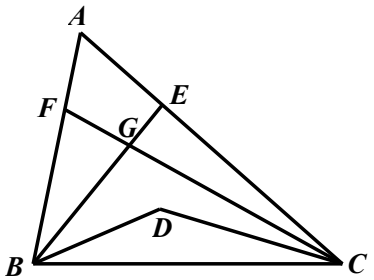
1. Do not open the booklet until you are told to do so.
2. Answer ALL 18 questions.
3. Write your answers in the answer sheet provided
4. No steps are needed to justify your answers.
5. Questions 1-4 are worth 5 marks each.
6. Questions 5-11 are worth 6 marks each.
7. Questions 12-17 are worth 8 marks each.
8. Questions 18 are worth 10 marks.
9. No marks will be deducted for wrong answers.
10. No marks will be given for unanswered questions.
11. No calculators or mathematical instruments are allowed.

Questions 1 to 4 are worth 5 marks each.

1. Let a and b be positive number satisfying $a \cdot b^2 = 3$ and $a^4 \cdot b^5 = 6$. Find the value of $a^7 \cdot b^8$.

2. What are the last four digits of the sum
$$1 + 12 + 123 + 1234 + 12345 + 123456 + 1234567 + 12345678 + 123456789 ?$$
Give your answer as a 4-digit number.

3. As shown in the figure, BE is the bisector of $\angle ABD$ and CF is the bisector of $\angle ACD$. If BE and CF intersect at G. If $\angle BDC = 140°$ and $\angle BGC = 100°$, what is the degree of $\angle A$?



4. Let n be a positive integer. If the highest common factor of n and 140 is 14, and the highest common factor of n and 66 is 6, what is the sum of the two smallest positive values of n?

Questions 5 to 11 are worth 6 marks each.

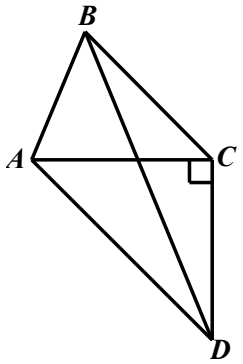
5. Let m and n be two positive integers satisfying

$$\frac{5}{6}-\frac{7}{12}+\frac{9}{20}-\frac{11}{30}+\frac{13}{42}-\frac{15}{56}+\frac{17}{72}=\frac{m}{n}.$$

Find the minimum value of $m+n$.

6. If a, b and c are integers satisfying $abc=240$, $ac+b=46$ and $a+bc=64$. Find the value of $a+b+c$.

7. In the quadrilateral $ABCD$, $\triangle ABC$ and $\triangle ACD$ are both isosceles triangles and point C is the common apex. Given that $\angle ACD = 90°$, $\angle ACB = 45°$ and $BD = 12$. Find the area of quadrilateral $ABCD$.



8. If $(4x-1)^4 = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4$, find the value of $a_0 - \frac{a_1}{2} + \frac{a_2}{2^2} - \frac{a_3}{2^3} + \frac{a_4}{2^4}$.

9. If $x - z = y - w = 26$, find the minimum value of $x^2 + y^2 + z^2 + w^2 - xy - yz - zw - wx$.

10. How many integers k are there such that the equation

$$\frac{(k+4)x^2+(k+2)x-8}{(k+1)x-2}=kx+1$$

has real number solutions?

11. There are nine robots numbered 1~9 standing in the 3 × 3 grid table, and each robot says one sentence. It is found that exactly one robot in each row tells lies, and exactly one robot in each column tells lies. Find the 3-digit number $\overline{def}$.

a: The numbers of the three robots in my column can be appropriately adjusted to form an arithmetic sequence;
b: The difference between the numbers of my left and right robots is 1;
c: I'm number 5;
d: There are two robots in my row telling lies;
e: My number is the smallest in my row;
f: The sum of the numbers of robots in my row is 21;
g: The sum of the numbers of two robots adjacent to me is 12;
h: The product of the three number of robots in my row is 6;
i: The numbers in my column are all even numbers.

a	***b***	***c***
d	***e***	***f***
g	***h***	***i***

Questions 12 to 17 are worth 8 marks each.

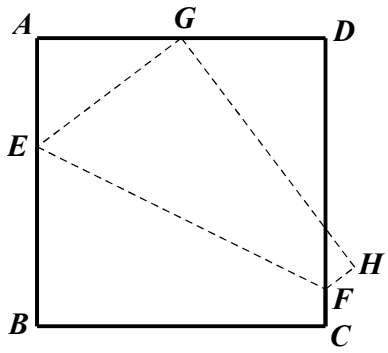
12. Find the value of $\sqrt{3(\sqrt{2}+\sqrt{3}+\sqrt{5})(3\sqrt{2}+2\sqrt{3}-\sqrt{30})}$.

13. Given that $a^2+4a+1=0$ and $\dfrac{a^4+ma^2+1}{2a^3+ma^2+2a}=-10$, find the value of m .

14. Let a, b and c be rational numbers satisfying $\begin{cases} a(b+c)+1=bc \\ b(c+a)-7=2ca \\ c(a+b)-4=4ab \end{cases}$. Find the value of $a^2+b^2+c^2$.

15. If n is an integer satisfying the value of $n^3-14n^2+56n-64$ is a prime number, find the value of n.

16. Let $ABCD$ be a square sheet of paper with $AB = 12$. If we fold the sheet of paper along the line EF, point B will coincide with G, the midpoint of AD as shown in the diagram. Find the area of the quadrilateral $BCFE$.



17. If p and q are two prime numbers such that $p^2 - 4q^4$ is a square number, how many such pairs of (p, q) can be found?

Questions 18 are worth 10 marks.

18. In your opinion, from question 1 to 17, your favourite question is question _____ and the most difficult question is question _____. (As long as your answer is within 1 to 17, you get full marks, otherwise you get zero.)