



2023 Spring Cup
Mathematical Olympiad
PRELIMINARY ROUND

Date: 28 January 2023
Time Given: 1 hour 30 minutes
Level: Primary 4
Name: _____

Instructions to Candidates

1. Do not open the booklet until you are told to do so.
2. Answer ALL 20 questions.
3. Write your answers in the answer sheet provided.
4. No steps are needed to justify your answers.
5. Questions 1-7 are worth 4 marks each.
6. Questions 8-14 are worth 6 marks each.
7. Questions 15-19 are worth 8 marks each.
8. Question 20 is worth 10 marks.
9. No marks will be deducted for wrong answers.
10. No marks will be given for unanswered questions.
11. No calculators or mathematical instruments are allowed.

Questions 1 to 7 are worth 4 marks each.

1. Calculate $1+2+3+\cdots+18+19+20+19+18+\cdots+3+2+1=$

【Solution】 $20 \times 20 = 400$

2. There are some monks in a mountain, and they need to draw water from the river. Two old monks can carry a bucket of water slung on a bamboo pole across their shoulders. Or, one young monk can carry two buckets of water slung on a bamboo pole across his shoulder. If 38 bamboo poles and 58 buckets are used, how many young monks are there? How many old monks?

【Solution】 Suppose there are only old monks, then one bucket matches one bamboo pole, and there are $58-38=20$ buckets left unmatched. If one bamboo pole is actually being used by a young monk carrying two buckets, and it is mistakenly counted as being shared by two old monks carrying only one bucket, there will be $2-1=1$ bucket unmatched. Therefore, there are $20 \div 1 = 20$ young monks each carrying two buckets. 20 bamboo poles correspond to two buckets while the other 18 bamboo poles correspond to one bucket, hence there are $18 \times 2 = 36$ old monks. The answer is 20 young monks and 36 old monks.

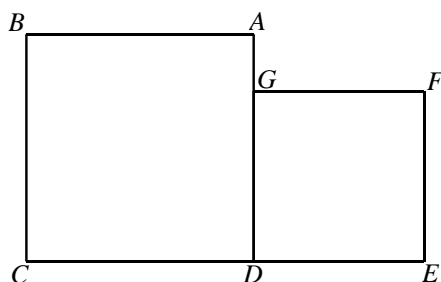
3. In the year-end examination, Frank's average mark of Chinese, Mathematics and Music Theory is 96. When the result of English exam is released, his overall average mark of four subjects is decreased by 2, what did Frank get for English?

【Solution】 The average mark for all 4 subjects is $96-2=94$, and an overall of $2 \times 3 = 6$ marks are taken away from Chinese, Mathematics and Music Theory to make up for English such that English can reach the average 94 marks. Hence, Frank's mark for English is $94-6=88$.

4. Anna, Blake and Cooper have 73 pieces of chocolate in total. If Cooper eats 3 pieces of chocolate, Blake and Cooper would have the same number of chocolates. If Blake gives Anna 2 pieces of chocolate, Anna's chocolate would be two times the amount of Blake. How many chocolates does Cooper have originally?

【Solution】 Cooper has 3 more pieces than Blake, so if Blake gives Anna 2 pieces, Cooper will have 5 more pieces than Blake. We can find currently Blake has $(73-5) \div (1+1+2) = 17$ pieces of chocolate. Hence, originally, Blake has $17+2=19$, and Cooper has $19+3=22$ pieces.

5. As shown in the figure, $ABCD$ and $DEFG$ are both squares. If $CE = 14$ and $AG = 2$, what is the sum of the area of these two squares.



【Solution】 The sum of the side length of both squares is 14, and the difference is 2. It can be found that the side length of square ABCD is $(14 + 2) \div 2 = 8$ and the side length of square DEFG is $(14 - 2) \div 2 = 6$, therefore the sum of two areas is $8 \times 8 + 6 \times 6 = 100$.

6. Four classes participated in a contest, Arthur and Benedict are guessing the outcome, they make the following predictions:

Arthur predicts: “3rd class will be the champion, 2nd class will be the first runner-up, 1st class will be the second runner-up and 4th class will be the third runner-up”

Benedict predicts: “2nd class will be the champion, 4th class will be the first runner-up, 3rd class will be the second runner-up and 1st class will be the third runner-up”

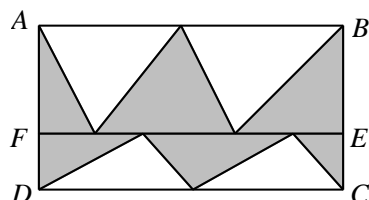
It turned out that only Benedict’s prediction on the “4th class will be the first runner-up” is true and all the other predictions made by them are wrong.

The correct ranking of the four classes should be: Champion: _____ class, first runner-up: _____ class, second runner-up: _____ class, third runner-up: _____.

【Solution】 A chart can be formed to find the answer.

	champion	First runner-up	Second runner-up	Third runner-up
1 st class	√	×	×	×
2 nd class	×	×	√	×
3 rd class	×	×	×	√
4 th class	×	√	×	×

7. As shown in the figure, the rectangle $AFEB$ and rectangle $FDCE$ combined to form another rectangle $ABCD$ with height 20 and width 12. The area of the shaded region is _____.



【Solution】 This is a typical Half-Area Model. In rectangle ABEF, the shaded part is half of ABEF. Likewise, in rectangle CDFE, the shaded part is also half of CDFE. Combining them together, we have the area of the shaded region is half of the big rectangle ABCD, and the answer is therefore $\frac{1}{2} \times 20 \times 12 = 120$.

Questions 8 to 14 are worth 6 marks each.

8. Starting with a certain number of cells, two cells will die every hour and each of the remaining cells will split into two. If there are 1284 cells after 8 hours, how many cells are there at the beginning?

【Solution】

$$\begin{array}{ccccccccccc}
 \text{after 8 hours} & & \text{after 7 hours} & & \text{after 6 hours} & & \text{after 5 hours} & & \text{after 4 hours} & & \text{after 3 hours} \\
 1284 & \xrightarrow{\div 2 + 2} & 644 & \xrightarrow{\div 2 + 2} & 324 & \xrightarrow{\div 2 + 2} & 164 & \xrightarrow{\div 2 + 2} & 84 & \xrightarrow{\div 2 + 2} & 44 \\
 & \xrightarrow{\div 2 + 2} & 24 & \xrightarrow{\div 2 + 2} & 14 & \xrightarrow{\div 2 + 2} & 9 & \xrightarrow{\div 2 + 2} & 5 & \xrightarrow{\div 2 + 2} & 3
 \end{array}$$

9. Jonah is writing down a sequence according to the following rule:

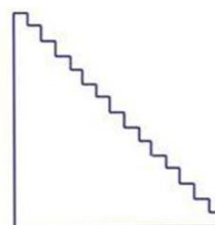
If he wrote down an even number, the next number is the current number divided by 2 and add 2.

If he wrote down an odd number, the next number is the current number multiplied with 2 and subtract 2.

By starting with the number 36, he wrote down: 36, 20, 12, 8, ..., what is the 2016th number in this sequence?

【Solution】 Continue writing this sequence: 36, 20, 12, 8, 6, 5, 8, 6, 5, 8... It is found that starting from the fourth number, every 3 numbers form a repeating pattern, which is 8, 6, 5. $2016 - 3 = 2013$, $2013 \div 3 = 671$, and hence the 2016th number in this sequence is 5.

10. The figure below depicts a staircase of a building from ground floor to first floor. If there are 15 steps in this staircase, each step is 16cm high with depth 26cm and the width of this staircase is 3m, how much money it would take to cover the staircase with carpet that costs 80 cents for every meter squared?

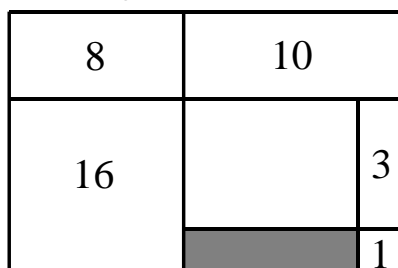


【Solution】 If we put the carpet on level ground, it would be a rectangle. This rectangle has a width of 3m, same as the width of the staircase. The length of this rectangle is the same as the length of the staircases combined, which is $(0.26 + 0.16) \times 15 = 6.3$ meters. Hence the carpet has an area of $6.3 \times 3 = 18.9 \text{ m}^2$, and the cost of the carpet is $18.9 \times 80 = 1512$ cents or \$15.12

11. If a 6-digit number $1992\square\square$ is divisible by 105, what are the last two digits?

【Solution】 First try 199200, $199200 \div 105 = 1897\text{R}15$, and this 15 is a surplus. A surplus of 15 is the same as a shortage of $105 - 15 = 90$. Hence we can add 90 to 199200 to obtain a multiple 105. The answer is 90.

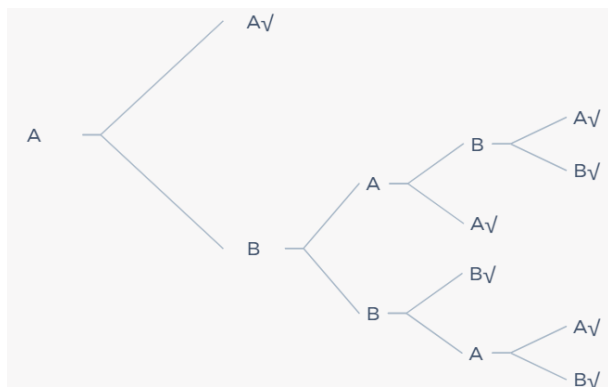
12. As shown in the figure, a big rectangle is divided into 7 smaller rectangles. The area of these small rectangles are 8, 10, 16, 3, 1 respectively, find the area of the shaded region.



【Solution】 The big rectangle at the bottom right corner has an area of $10 \div 8 \times 16 = 20$. $20 - 3 - 1 = 16$, hence the shaded area plus the blank area is 16. The blank area is triple the shaded area, and hence the shaded area is $16 \div (1 + 3) = 4$.

13. Adam and Bradley are playing table tennis, whoever wins the first two games consecutively is the winner. If no one wins the first two games consecutively, whoever wins three games (not necessarily consecutively) is the winner. If they play until there is a winner, how many ways there are?

【Solution】 As shown below, we can consider if Adam wins the first game.



There are 7 possible ways. Likewise, if Bradley wins the first game, there will be another 7 ways. Altogether, there are 14 ways.

14. Tony and Steve are driving towards each other and they depart from Point A and B respectively at the same time. 2 hours after their departure, they are 141km apart. If they meet each other 5 hours after their departure, what is the distance between point A and B?

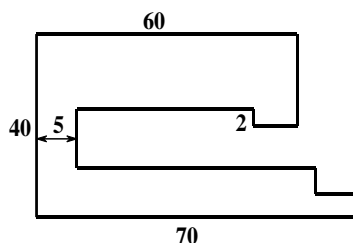
【Solution】 Duration = sum of distance \div sum of speed. In this question, it takes 5 hours for the two people to complete the entire distance AB, and it takes 3 hours for them two to complete 141km. Hence, the speed's sum is $141 \div (5 - 2) = 47$ km/h, and distance AB is $47 \times 5 = 235$ km.

Questions 15 to 19 are worth 8 marks each.

15. Given that $\overline{abcd} + \overline{abc} + \overline{ab} + a = 1370$, find \overline{abcd} .

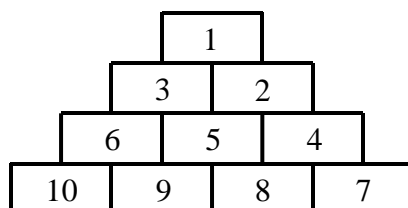
【Solution】 The equation can be rewritten as $1111a + 111b + 11c + d = 1370$, and hence $a = 1$, resulting in $111b + 11c + d = 1370 - 1111 = 259$. Then the only possible value for b is 2, and hence it can be found that $c = 3$, $d = 4$. The answer is $\overline{abcd} = 1234$.

16. What is the perimeter of the following shape?

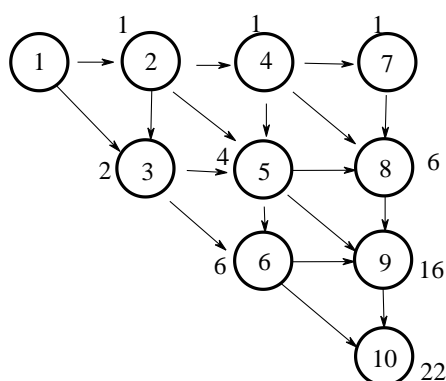


【Solution】 All the edges facing left and all the edges facing right, have the same overall length, which is $40 + 2 = 42$ m. Likewise, all the edges facing up, and all the edges facing down, have the same overall length, which is $70 + (60 - 5) = 125$ m. Therefore, the perimeter is $[70 + (60 - 5)] \times 2 + (40 + 2) \times 2 = 334$ m.

17. As shown in the figure, there are ten rooms labelled with the numbers 1 to 10. A person is only allowed to move to an adjacent room whose labelled number is greater than his current room. How many possible paths are there to travel from room number 1 to room number 10?



【Solution】 Using the Marking-Number Method, The answer can be found to be 22.



18. The monkey king is distributing peaches. If the older monkeys receive 5 peaches each and the younger monkeys receive 3 peaches each, the monkey king can keep the remaining 10. If the older monkeys and the younger monkeys each receive 4, the monkey king can keep the remaining 20. The number of older monkeys is _____ more than the number of younger monkeys.

【Solution】 “If the older monkeys receive 5 peaches each and the younger monkeys receive 3 peaches each, the monkey king can keep the remaining 10. If the older monkeys and the younger monkeys each receive 4, the monkey king can keep the remaining 20.” This implies that after the older monkeys each receive 5 and the younger each receive 3, if each older monkey gives away 1 peach and at the same time, each young monkey receives one more peach, there will be $20 - 10 = 10$ more peaches. This shows that the number of older monkeys is 10 more than the number of younger monkeys.

19. Sunflowers, Lily flowers and Orchid flowers are planted in a garden. It is known that:

1. There is only one day in a week, three flowers can be found bloom together.
2. All three flowers cannot bloom for more than three consecutive days.
3. In a week, any two flowers will not bloom together for at most one day.
4. Sunflowers do not bloom on Tuesday, Thursday and Sunday.
5. Lily flowers do not bloom on Thursday and Saturday.
6. Orchid flowers do not bloom on Sunday.

On which day of the week, we can find three flowers bloom together?

【Solution】 As shown in the figure below, we mark the day with “×” if on that day, this flower does not bloom; otherwise we mark the day with “√”. First we mark according to condition 4, 5, 6. After that, according to condition 3, it is known that Sunflower blooms on Saturday, Lily flower

blooms on Tuesday and Sunday, and Orchid flower blooms on Tuesday and Thursday. Given condition 2, it is known that Lily does not bloom on Monday, Orchid does not bloom on Wednesday. Hence it is only possible that all three flowers bloom together on Friday.

Day of the week Flower	1	2	3	4	5	6	7
Sunflower		×		×		√	×
Lily flower	×	√		×		×	√
Orchid flower		√	×	√			×

Questions 20 is worth 10 marks.

20. In your opinion, from question 1 to 19, your favourite question is question _____ and the most difficult question is question _____.

(As long as your answer is within 1 to 19, you get full marks, otherwise you get zero.)