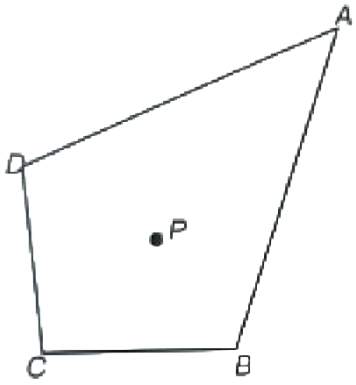
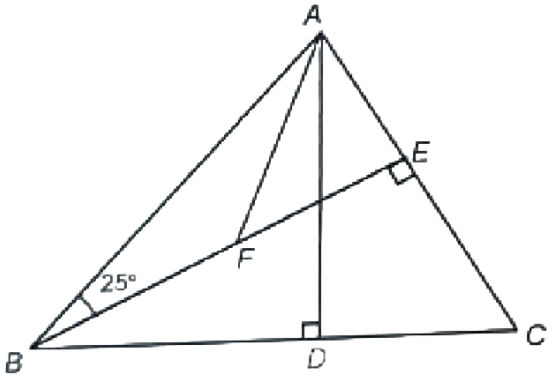
NMOS 2013 Special Round

1. The length of a rectangle is increased by 25% and its width is decreased by p%. Given that the area of the rectangle remains unchanged, find the value of p.

2. A quadrilateral $ABCD$ with an interior point P is shown below. Given that the perimeter of the quadrilateral $ABCD$ is 40 cm and the shortest distance from point P to any of the four sides is 5 cm, find the area in cm^2, of the quadrilateral $ABCD$.



3. In the diagram below, ABC is a triangle. AD $\perp$ BC and BE $\perp$ AC. The point F is on BE such that $\angle$BAF = $\angle$DAF. If AF = BF and $\angle$ABE = 25°, find $\angle$ACB in degrees.



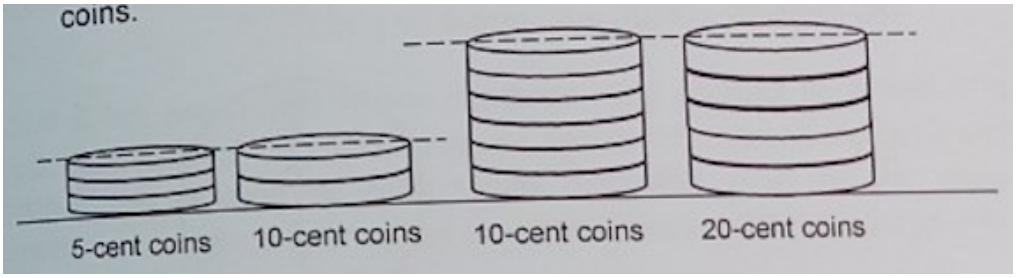
4. Note that

$$1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{2}}}=1+\cfrac{1}{1+\cfrac{4}{9}}=1+\frac{9}{13}=\frac{22}{13}$$

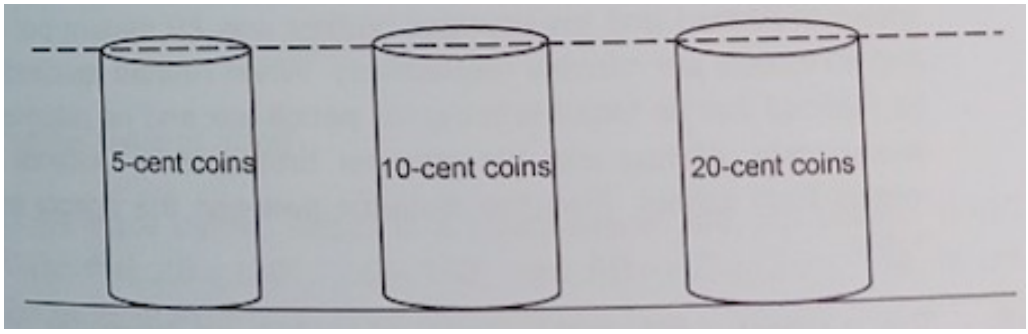
It is known that the fraction $\frac{13}{10}$ can be written as $1+\cfrac{1}{a+\cfrac{1}{b+\cfrac{1}{c}}}$, where a, b and c are whole numbers. Find the value of $a + 2b + 3c$.

5. It is known that for a particular country,

   (i) The height of three 5-cent coins is the same as that of two 10-cent coins;
   (ii) The height of six 10-cent coins is the same as that of five 20-cent coins.



There are three cylinders, of the same height, built with 5-cent, 10-cent and 20-cent coins respectively. If the total value of the coins is 1640 cents, what is the total number of coins used?



6. A 2013-digit number is formed by writing the whole numbers starting from 1, in a connected way, as follows:

   123456789101112…

   What are the last four digits of this number?

7. Today is 20 July 2013. We can record the date as 20130720, and this number is divisible by 3 since $20130720 = 6710240 \times 3$. For the date 22 July 2013, the corresponding number is 20130722, and this number is divisible by 2. If we record the other dates in the same way, how many days are there altogether in this year where the corresponding number is divisible by 3 or by 2?

   (Note that there are 31 days in January, March, May, July, August, October and December; while there are 30 days in April, June, September and November. There are only 28 days in February of 2013.)

8. How many times in a day is the angle between the hour hand and the minute hand 180 degrees?

   (For example, the angle between the hour hand and the minute hand is 180 degrees at 6:00 a.m. and 6:00 p.m.)

9. Ahmad and his younger brother studied at the same school. One morning, they walked from home to their school at the same time. The speed of Ahmad and his younger brother was 80 meters per minute and 50 meters per minute respectively. When Ahmad reached school, he realized that he forgot to bring his pencil box and returned home immediately. Ahmad met his younger brother at a distance of 210 meters from school. Find the distance between the school and their home.

10. The numbers 1461, 5961, 3827 and 4201 are examples of 4-digit numbers containing the digit 1 or the digit 2 or both. How many 4-digit numbers contain the digit 1 or the digit 2 or both?

11. Nine numbers are filled in a 3 × 3 array such that the sum of the numbers in each row, column and diagonal is the same. This sum is called the "common sum". For example, the common sum in Figure 1 is 15. In figure 2, the numbers 7, 67 and 139 had been filled in.
Find the common sum of Figure 2.

2	9	4
7	5	3
6	1	8

Figure 1

	139	
		67
	7	

Figure 2

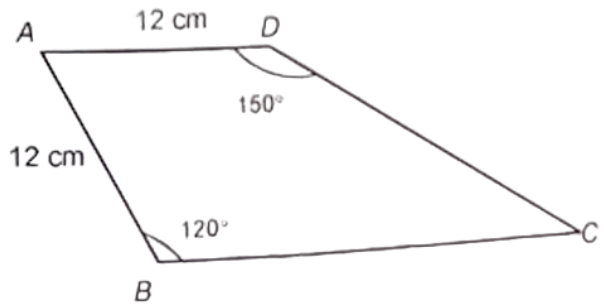
12. A and B are two 2-digit whole numbers. The product of A and B is $A \times B$. Abel read all the digits correctly, except for the unit digit of A and obtained the product as 803, while Benjamin read all the digits correctly, except for the tens digit of A and obtained the product as 165.
Find the correct value of $A \times B$.

13. The first digit of the 5-digit number $\overline{2abcd}$ is 2. If this first digit is moved to the last place, we will obtain a new 5-digit number $\overline{abcd2}$. Given that

$$\overline{2abcd} : \overline{abcd2} = 5 : 9$$

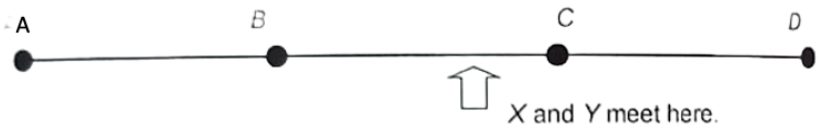
find the value of $\overline{abcd}$.

14. In the figure below, $ABCD$ is a quadrilateral, with AD parallel to BC. Given that $\angle B = 120°$, $\angle D = 150°$ and $AB = AD = 12$ cm, find, in cm, the length of BC.

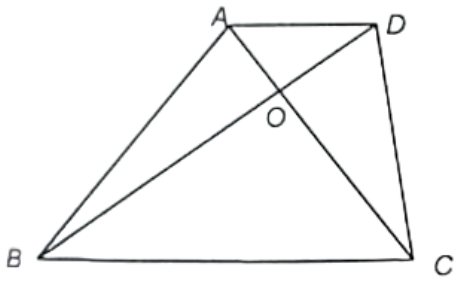


15. If two numbers x and y, where $x \leq y$, are removed from 1, 2, 3, …, 100, then the average of the remaining numbers is an integer. How any possible pairs of x and y are there?

16. There are 4 strain stations A, B, C and D where each station is equally far away from the other stations. One day, train X travels from station A towards station D while train Y travels from station D towards station A at the same time. Train X is faster than train Y by 40%. If train X meets train Y at a distance of 18 km from station C, what is the distance between station A and station D?

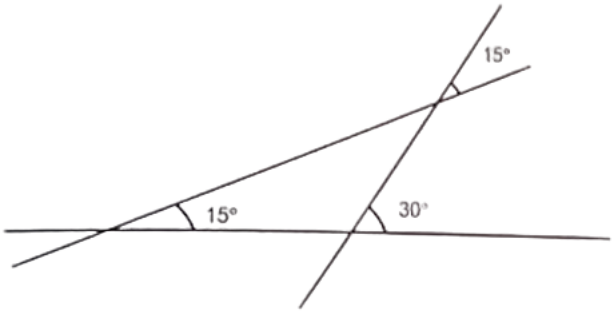


17. In the figure below, $ABCD$ is quadrilateral with AD parallel BC and $BO = 3OD$. The diagonals AC and BD intersect at O. If the area of $ABCD$ is 240 cm^2, find, in cm^2, the area of the triangle AOB.



18. There are two types of pens, Type A and Type B. The prices of these pens are whole numbers in dollars and Type A is more expensive than Type B. It costs 17 dollars to buy one Type A pen and one Type B pen. Daryl has 35 dollars and he wants to use all his money to buy these pens (it may be 2 types of pens or just one type of pen). He find that no matter how he allocates his money, he always cannot spend all 35 dollars. Find the price of a Type A pen in dollars.

19. The figure below shows 3 lines such that any 2 lines are intersecting each other. The acute angles between any two intersecting lines are shown.



There are n lines such that any 2 lines are intersecting each other. If the acute angle between any two intersection liens must be a multiple of 15°, find the largest possible value of n.

20. Given that the 4-digit number $\overline{abcd}$ satisfies both conditions (i) and (ii), as follows:

(i) $a + b + c + d = \overline{ab}$
(ii) $a \times b \times c \times d = \overline{cd}$

find the value of $\overline{abcd}$.

(Examples of 4-digit numbers which satisfy (i): 1745 or 1754 as 1 + 7 + 4 + 5 = 17.)