

## 2017 SMOPS Questions

1. Calculate:  $(12345 + 23451 + 34512 + 45123 + 51234) \div 5$ .

2. Given that  $a, b, c, d$  are positive numbers and  $a < b < c < d$ , which of the following expressions has the largest value?

(1)  $a \times (b + c + d)$

(2)  $b \times (a + c + d)$

(3)  $c \times (a + b + d)$

(4)  $d \times (a + b + c)$

3. Jason drinks 60% of the water in a bottle and then refills 100 ml. Now the amount of water in the bottle is half of the initial amount. Find the initial volume (in ml) of water in the bottle.

4. Given that  $a, b, c$  are prime numbers and  $31 + a = 26 + b = 20 + c$ . Find the value of  $a \times b \times c$ .

5. A class of 20 students sits in a circle. They are numbered from 1 to 20 in a clockwise order. Now every third student will leave the circle, starting with students numbered 3, 6, 9 and so on. This process continues until there is only one student remaining. What is the number of the last remaining student?

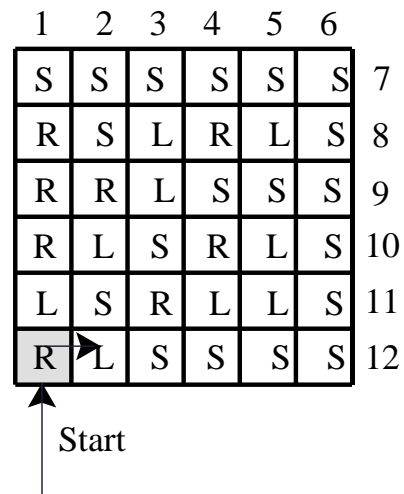
6. The average weight of the people in a group was calculated. Aaron, who weighs 45 kg, joined the group and the average changed to 61 kg. Then Ben, who weighs 71 kg, joined the group after Aaron. The average changed to 62 kg. What was the average weight of the group before Aaron and Ben joined them?

7. A twelve-digit even number  $\overline{123A456A789A}$  is divisible by 9 but not divisible by 5. Find the value of  $A$ .

8. In the year 2017, Jamie's age is equal to the sum of digits of the year in which she was born. If she was born before the year 2000, how old is she this year?

9. A 3-digit prime number is made of three distinct digits. The unit digit is equal to the sum of the other two digits. Find the unit digit.

10. Bryan enters a  $6 \times 6$  maze from the shaded cell as shown. Each cell is labelled L for turning left, R for turning right and S for going straight. He moves about the maze according to the label. For example, upon entering the first cell, he turns right, as shown in the diagram below. The process continues until he gets out of the maze. Among the cells labelled 1 to 12, which cell would Bryan exit the maze?



11. It is given that  $x, y, z$  are three positive integers smaller than 10. When the product of any two numbers is divided by the third number, the remainder is always 1. Find the value of  $x + y + z$ .

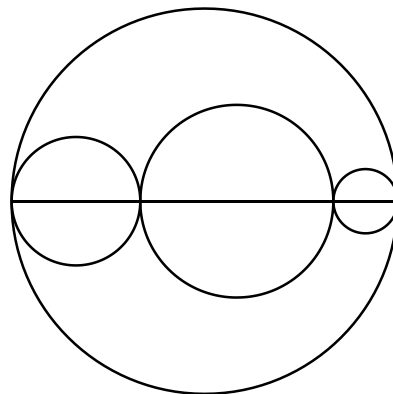
12. In the year 2018, the 1<sup>st</sup> of January is a Monday. If there are five Mondays in a month, we call it a “Mondayful” month. How many “Mondayful” months are there in 2018?

13. Bryan is the owner of two farms. One day, he places an order to purchase 8 robots for Farm A and 6 robots for Farm B. The robot factory has 14 robots available – 10 in warehouse C and 4 in warehouse D. The shipping cost (per robot) from warehouse to farm is shown in the table below.

Shipping cost (per robot)	To Farm A	To Farm B
From Warehouse C	\$800	\$400
From Warehouse D	\$500	\$300

What is the minimum transportation cost (in dollars) to fulfill Bryan's order?

14. In the diagram below, the circumference of the largest circle is 28. There are three smaller circles with centres lying on the diameter of the largest circle. The circles touch one another as shown in the diagram below. Find the sum of the circumference of the three smaller circles.  $\left( \pi = \frac{22}{7} \right)$



15. If  $x, y, z$  are all positive integers, find the number of solutions to the equation

$$x + y + z = 7$$

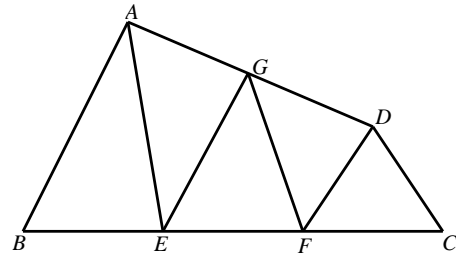
Note that order of unknowns is important. For example,  $(x = 1, y = 5, z = 1)$  and  $(x = 1, y = 5, z = 1)$  are considered two different solutions.

16. There are 64 small wooden cubes with side length 1 cm. 34 cubes have all six faces painted white while the other 30 cubes have all faces painted black. A  $4 \times 4 \times 4$  (in cm) large cube is formed using all the small cubes. What is the least possible area (in  $2 \text{ cm}^2$ ) of the black region on the surface of the large cube?

17. Using letters  $a, a, b, b, c, c$  how many ways are there to fill out a  $3 \times 2$  table (as shown in the diagram below) such that there are no identical letters on each row or each column?


18. There were 20 different toys in a box. Three children took turns to take the toys out to play and returned them back to the box. It is given that the children played with 11, 17 and 16 toys respectively. What is the least possible number of toys that were played by all three children?

19. In the diagram below,  $ABCD$  is a quadrilateral. Points  $E$  and  $F$  lie on  $BC$  such that  $BE = EF = FC$ . Point  $G$  is on  $AD$  such that  $GE$  is parallel to  $AB$ , and  $FG$  is parallel to  $CD$ . If the area of  $\triangle AEG$  is 9, find the area of the four-sided figure  $AEFD$ .



20. What is the least number of cuts needed to cut a piece of  $5 \times 7$  paper into 35 pieces of  $1 \times 1$  unit squares? Assume that stacking is allowed when cutting the paper.

21. In the following expression, each letter in  $A, B, C, D, E, F, G, H, I, J$  represents a distinct digit.

$$A + \overline{BC} + \overline{DEF} + \overline{GHIJ} = X$$

Among all possible values of the sum  $X$ , which value is the closest to 2010?

( $\overline{BC}$  denotes a 2-digit number where the tens digit is  $B$  and the unit digit is  $C$ .)

22. In a 12-hours duration, how many times do the hour and minute hand form an angle of  $30^\circ$  ?

23. Three pirates had a gambling session on an island. Before the session, their money was in the ratio  $7:6:5$ . After the game, their money was in the ratio  $6:5:4$  (in the same order). If one of them won 8 dollars, how much money (in dollars) did they have in total?

24. A rabbit and a tortoise entered a 10000 metres race. The rabbit runs at a speed 5 times as fast as the tortoise. They started at the same instant from the starting line. During the race, the rabbit took a nap while the tortoise ran continuously. When the rabbit woke up, he was 4000 metres behind the tortoise. In the end, the rabbit was 200 metres away from the finishing line when the tortoise won the race. What is the distance (in metres) covered by the tortoise while the rabbit was sleeping?



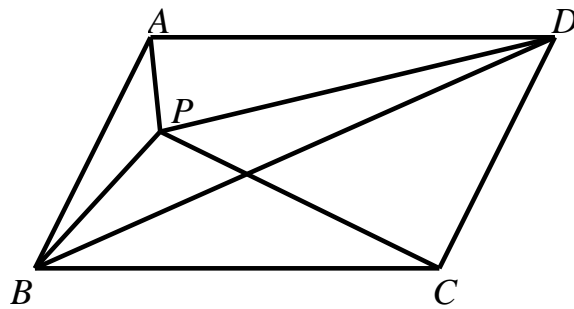
25. Many fractions can be expressed as recurring decimals.

For example,  $\frac{26}{111} = 0.234234234\ldots = 0.\overline{234}$

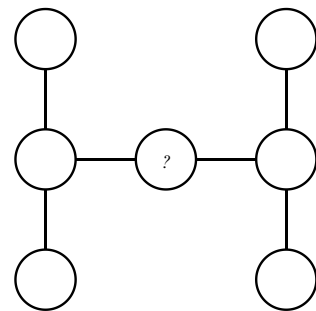
It is known that  $\frac{1}{19}$  can be expressed as a recurring decimal  $0.\overline{a_1 a_2 a_3 \ldots a_{17} a_{18}}$ .

Find the value of  $a_1 + a_2 + a_3 + \ldots + a_{17} + a_{18}$ .

26. In the diagram below, point  $P$  is inside parallelogram  $ABCD$ . If the area of  $\triangle ABP$  and  $\triangle BPC$  is 75 and 124 respectively, find the area of  $\triangle BPD$ .



27. Using seven distinct integers from 1 to 9 to fill out the circles in the diagram below such that the products of the three numbers along each straight line are equal, which number is in the centre circle?



$$\begin{array}{ccccccc} & & & & 2 & & \\ & & & & 4 & 6 & 8 \\ & & 10 & 12 & 14 & 16 & 18 \\ 20 & 22 & 24 & 26 & 28 & 30 & 32 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{array}$$