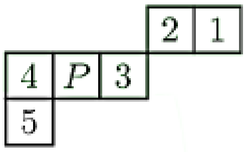
APMOPS 2011 Round 1

Time Duration: 2 hours Name: ___________ Marks: ___________

1. If the following figure is folded into the shape of a cube, what is the number opposite on the face marked P?



2. Find the value of

$$\left(1-\tfrac{1}{2}\right)\times\left(1-\tfrac{1}{3}\right)\times\left(1-\tfrac{1}{4}\right)\times\ldots\left(1-\tfrac{1}{2010}\right)\left(1-\tfrac{1}{2011}\right).$$

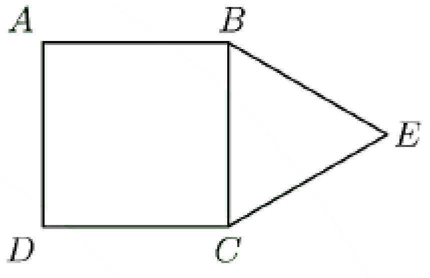
3. A circle of radius 1 m has some point lying on its circumference. Find the minimum number of point such that at least two points are less than 1 m apart.

4. Three sides of a four-sided figure are of lengths 4 cm, 9 cm and 14 cm respectively. If the largest possible length of the fourth side is x cm where x is a whole number, find the value of x.

5. Find the largest prime number that divides the number

$$(1\times 2\times 3\times \ldots \times 97\times 98)+(1\times 2\times 3\times \ldots \times 98\times 99\times 100).$$

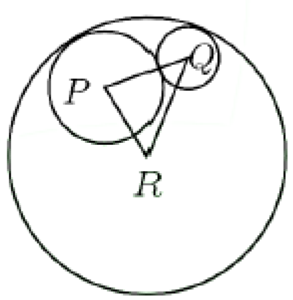
6. ABCD is a square and BCE is an equilateral triangle. If BC is 8 cm, find the radius of the circle passing through A, E and D in cm.



7. Find the value of

$$\frac{19}{20}+\frac{1919}{2020}+\frac{191919}{202020}+\dots+\frac{\underbrace{1919\dots19}_{2011\text{ of }19\text{'s}}}{\underbrace{2020\dots20}_{2011\text{ of }20\text{'s}}}.$$

8. The following diagram show a circle of radius 8 cm with the centre R. Two smaller circles with centres P and Q touch the circle with centre R and each other as shown in the diagram. Find the perimeter of the triangle PQR in cm.

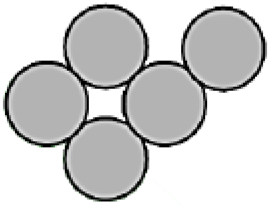


9. Peter and Jane competed in a 5000 m race. Peter's speed was 4 times that of Jane's. Jane ran from the beginning to the end, whereas Peter stopped running every now and then. When Jane crossed the finish line, Peter was 100 m behind. Jane ran a total of x m during the time Peter was not running. Find the value of x.

10. If numbers are arranged in three rows A, B, and C in the following manner, which row will contain the number 1000?

A	1	6	7	12	13	18	19	...
B	2	5	8	11	14	17	20	...
C	3	4	9	10	15	16	21	...

11. The following diagram shows 5 identical circles. How many different straight cuts are there so that the five shaded circles can be divided into two parts of equal areas?



12. A test with a maximum mark of 10 was administered to a class. Some of the results are shown in the table below. It is know that the average mark of those scoring more than 3 is 7 while the average mark of those getting below 8 is 5. Given that none scored zero, find the number of pupils in the class.

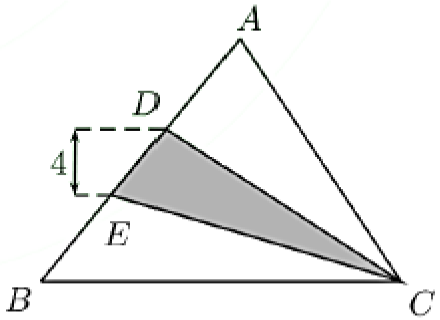
Score	1	2	3	...	8	9	10
Number of pupils	1	3	6	...	4	6	3

13. A test comprises 10 true or false questions. Find the least number of answer scripts required to ensure that there are at least 2 scripts with identical answer to all the 10 questions.

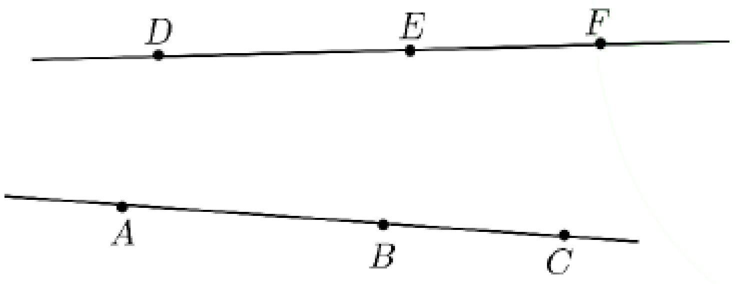
14. P_η is defined as the product of the digits in the whole number η. For example, $P_{19} = 1 \times 9 = 9$, $P_{32} = 3 \times 2 = 6$. Find the value of

$$P_{10} + P_{11} + P_{12} + \ldots + P_{98} + P_{99} .$$

15. ABC is a triangle with BC = 8 cm. D and E line on AB such that the vertical distance between D and E is 4 cm. Find the area of shaded region C D E in cm^2.

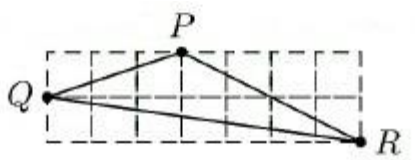


16. The points A, B, C, D, E and F are on the two straight lines as shown. How many triangles can there be formed with any 3 of the 6 points as vertices?

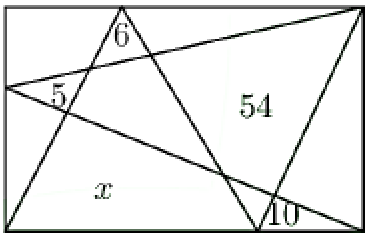


17. A group of 50 girls were interviewed to find out how many books they had borrowed from the school library in April. The total number of books borrowed by the girls in April was 88, and 18 girls had borrowed only 1 book each. If each girl had borrowed either 1, 2, or 3 books, find the number of girls that had borrowed 2 books each.

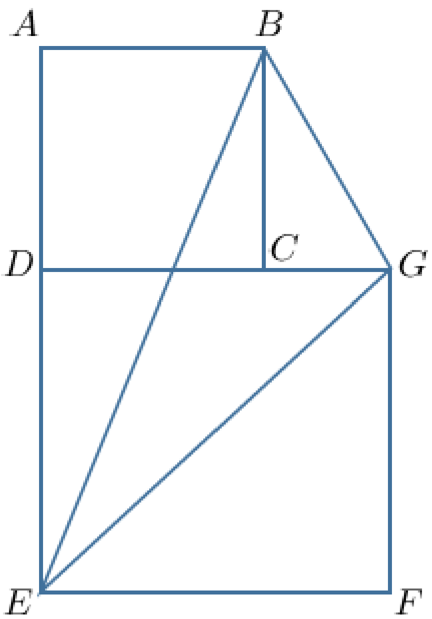
18. The following diagram shows a triangle PQR on a 2 by 7 rectangular grid. Find the sum of the angle of PQR and PRQ in degrees.



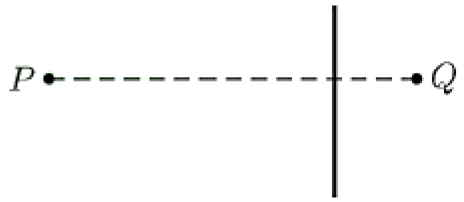
19. The following diagram shows a rectangle where the areas of the shaded regions are 5 cm^2, 6 cm^2, 10 cm^2, 54 cm^2 and x cm^2 respectively. Find the value of x.



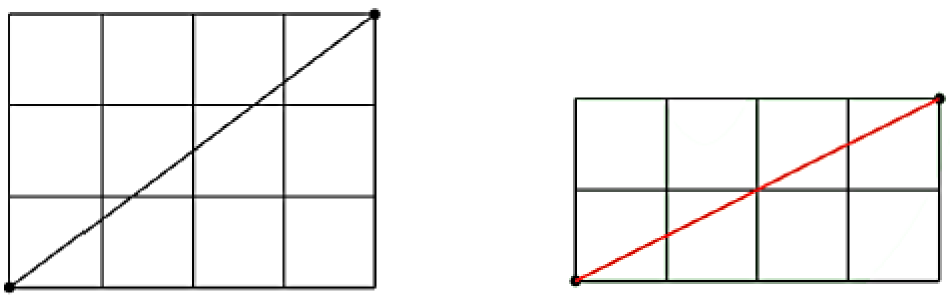
20. The following diagram shows two squares ABCD and DGFE. The side CD touches the side DG. If the area of D E F G is 80 cm^2, find the area of the triangle BGE in cm^2.



21. Two points P and Q are 11cm apart. A line perpendicular to the line PQ is 7cm from P and 4cm from Q. How many more lines, on the same plane, are 7 cm from P and 4 cm from Q?



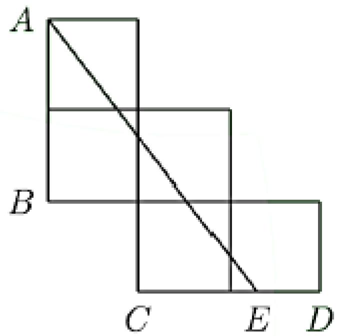
22. The greatest number of points of intersection of the grid lines that the diagonal of a rectangle with area 12 cm^2 can pass through is 3 as shown.



Find the greatest possible number of points of intersection that the diagonal of a rectangle with area 432 cm^2 can pass through.

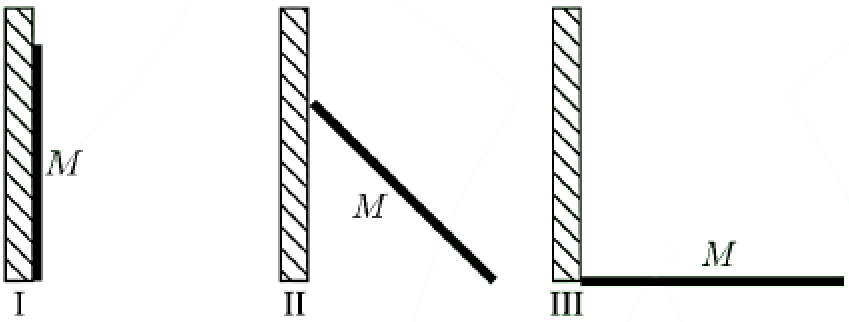
23. Given that (100 × a + 10 × b + c) × (a + b + c) = 1926 where a, b and c are whole numbers, find the value of a + b + c.

24. The following figure comprises 5 identical squares each of area 16 cm^2. A, B, C and D are vertices of the squares. E lies on CD such that AE divides the 5 squares into two parts of equal areas. Find the length of CE in cm.



25. Whole numbers from 1 to 10 are separated into two groups, each comprising 5 numbers such that the product of all the numbers in one group is divisible by the product of all the numbers in the other. If η is the quotient of such a division, find the least possible value of η.

26. Diagram Ⅰ shows a ladder of length 4 m leaning vertically against a wall. It slides down without slipping to Ⅱ, and then finally to a horizontal position as shown in Ⅲ. If M is at the mid-point of the ladder, find the distance travelled by M during the slide in m.



27. A theme park issues entrance tickets bearing 5-digit serial numbers from 00000 to 99999. If any adjacent numbers in the serial numbers differ by 5 (for example 12493), customers holding such a ticket could use the ticket to redeem a free drink. Find the number of tickets that have serial numbers with this property.

28. S_η is defined as the sum of the digits in the whole number η. For example, $S_3 = 3$ and $S_{29} = 2+9=11$. Find the value of

$$S_1 + S_2 + S_3 + ... + S_{2010} + S_{2011}.$$

29. Anthony, Benjamin and Cain were interviewed to find out how many hours they spend on the computer in a day. They gave the following replies.

- **Anthony:**
  I spend 4 hours on the computer.
  I spend 3 hours on the computer less than Benjamin.
  I spend 2 hours on the computer less than Cain.

- **Benjamin:**
  Cain spends 5 hours on the computer.
  The time I spend on the computer differs from Cain's time by 2 hours.
  The time I spend on the computer is not the least among the three of us.

- **Cain:**
  I spend more time on the computer than Anthony.
  I spend 4 hours on the computer.
  Benjamin spends 3 hours on the computer more than Anthony.

If only two of the three statements made by each boy are true, find the number of hours that Anthony spends on the company in a day.

30. ABC is a triangle and D lies on AC such that AD = BD = BC. If all the three interior angles of triangle ABC, measured in degrees, are whole numbers, find the greatest possible value of angle ABC in degrees.