

APMOPS 2013 Round 1

Time Duration: 2 hours

Name: _____

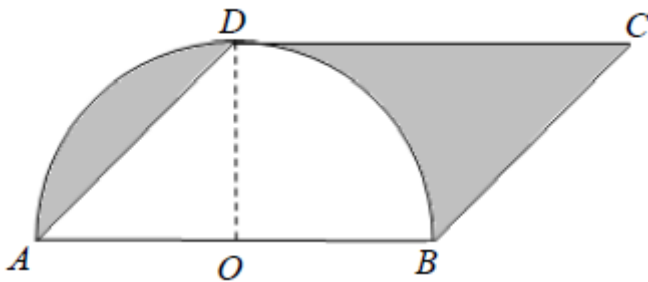
Marks: _____

1. Find the last 5 digit of the sum.

$$127354 + 27354 + 7354 + 354 + 54 + 4.$$

2. Find the sum of all two-digit numbers whose units digit and tens digit are both even.

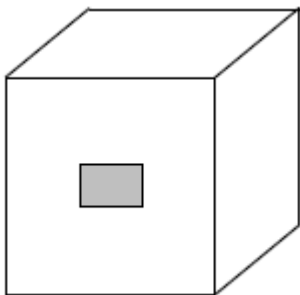
3. The diagram shows a semi-circle with centre O overlapped with a parallelogram $ABCD$. The diameter, AB , of the semi-circle is 12cm. Find the total area of the shaded regions in cm^2 .



4. Abel, Ben and Charlie took part in SMOPS 2012, which comprised 30 questions. They had correctly answered 26, 23 and 18 questions respectively. What is the least possible number of questions that were answered correctly by all 3 students?

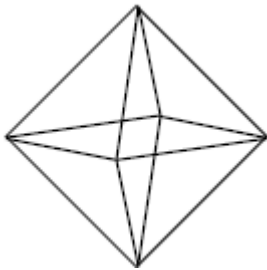
5. Find the value of $555 \times 554555 - 554 \times 555554$.

6. The diagram shows a cube with side length 5 cm. If a rectangular tunnel with dimensions 2 cm by 3 cm is made in the middle of the cube, find the amount of increase in the total surface area of the resulting solid in cm^2 .

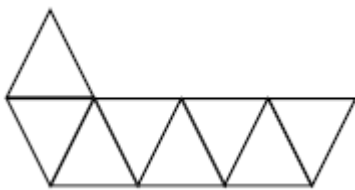


7. The average value of four whole numbers \overline{a} , $\overline{b5}$, $\overline{c17}$ and $\overline{d432}$, where a , b , c and d each represents the first digit of a number, is 1735. Find the value of $a + b + c + d$.

8. The diagram shows a regular octahedron, which is a solid composed of eight equilateral triangles.



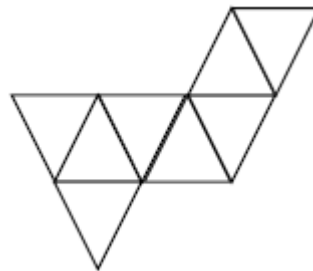
Which of the following patterns can be folded into a regular octahedron?



Pattern (1)



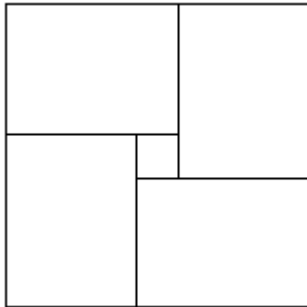
Pattern (2)



Pattern (3)

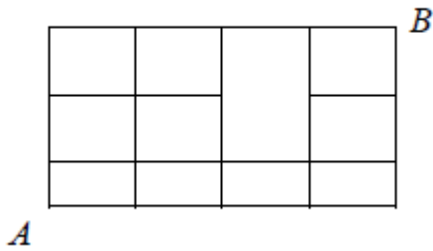
9. The sum of 2 prime numbers is equal to 2013. Find the product of these two numbers.

10. The figure shows a big square, which is divided into four identical rectangles and one small square. Given that the area of the small square is 16 cm^2 , the area of each rectangle is 140 cm^2 , find the width of each rectangle in cm.

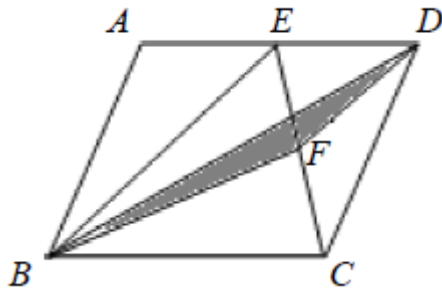


11. A 5-digit number written in the form $\overline{24abc}$ has the last three digits unknown. If this number is divisible by 3, 4 and 5 respectively, find the greatest possible value that \overline{abc} can take.

12. In the diagram, an ant is moving from A to B. If the ant is only allowed to move to the right or upwards along the grid lines, how many different paths are there from A to B?



13. The diagram shows a parallelogram ABCD. E is the midpoint of AD. F is the midpoint of EC. If the area of the triangle BFD is 9 cm^2 , find the area of the parallelogram ABCD in cm^2 .



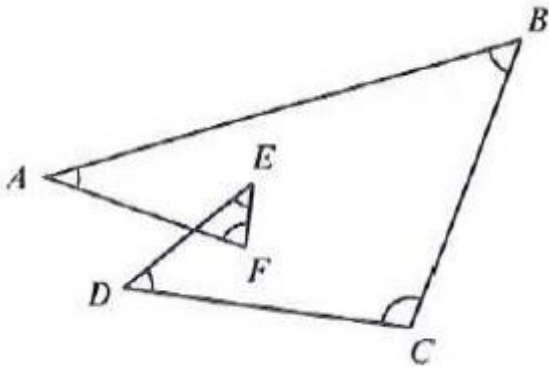
2013

14. If we write $\frac{2013}{1990}$ in the form

$$a + \frac{1}{b + \frac{1}{c + \frac{1}{d + \frac{1}{e}}}}$$

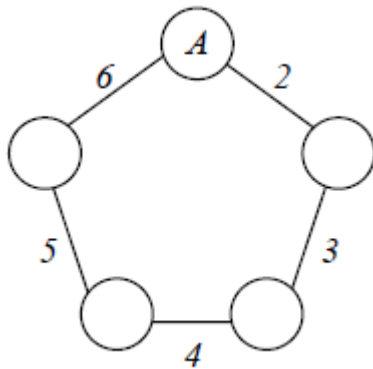
where a, b, c, d and e are positive integers, what is the value of $a + b + c + d + e$?

15. In the given diagram, find the value (in degrees) of
 $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F$



16. In how many different ways can four children share 8 identical chocolates so that each child get at least one?

17. In the given diagram, each circle contains a natural number and the diagram satisfies the following conditions:

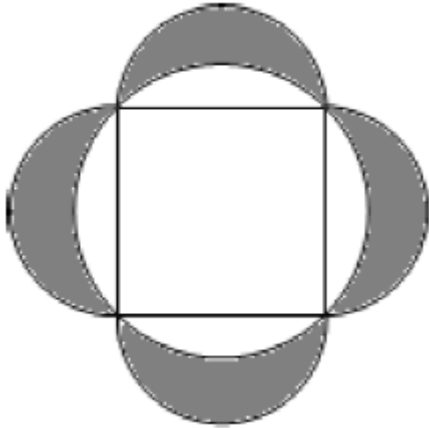


- i. The number labelled along each edge represents the difference between the numbers in the two circles joined by the edge.
- ii. The sum of the numbers in the 5 circles is equal to 1979.

Find the number in circle A.

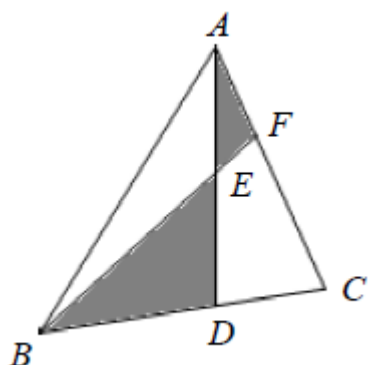
18. A certain type of water bottle is sold at \$10 in both Store A and Store B. Mrs. Lim would like to buy a few water bottles for a Children's Home. Store A sells the water bottle with an offer of "Buy 5 Get 1 Free" (no free bottles for buying fewer than 5 water bottles); store B gives a 15% discount for customers who buy 4 or more water bottles. What is the least amount of money (in \$) that Mrs. Lim needs to spend in order to get 14 water bottles?

19. A square of side length 18 cm is inscribed in a circle. Semi-circles are constructed on its sides, as shown in the diagram. Find the total area of the four shaded lunes in cm^2 .



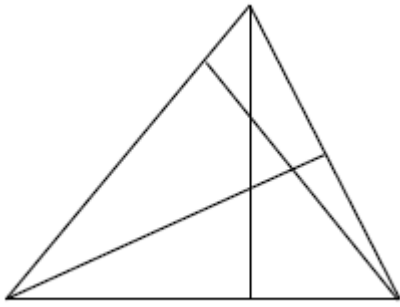
20. Four teams participated in a soccer tournament. Each team played against all other teams once each. 3 points were awarded for a win, 1 point for a draw and 0 points for a loss. At the end of the tournament, the four teams have obtained 5, 1, x , 6 points respectively. Find the value of x .

21. In the diagram, the area of triangle ABC is 40. Given that $2BD = 3CD$ and $AE = DE$, find the area of the shaded region.



22. If integers are selected randomly from 1 to 35, what is the minimum number of integers that need to be selected such that among the chosen numbers we can always find two integers whose difference is divisible by 7?
23. A team of workers are sent to two construction sites A and B respectively. The amount of work to be done at construction site A is 50% more than that at construction site B. In the morning, the number of workers sent to construction site A is 3 times the number of workers sent to site B. In the afternoon, the ratio of workers sent to construction site A and B is 7: 5. By the end of the day, the work at construction site A is fully completed, while construction site B still requires 8 workers to work for another full day. Assuming the workers work at the same rate, find the total number of workers in this team.

24. Find the total number of triangles in the figure below.

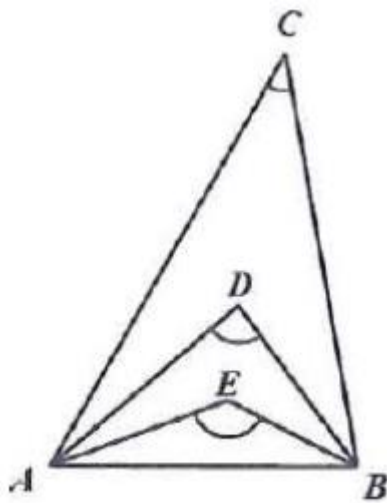


25. A car and a motorcycle started travelling towards each other at the same instant, from cities A and B respectively. 72 minutes later, they met along the road and continued to travel towards their destinations. Given that the speed of the car is $1\frac{1}{3}$ times that of the motorcycle, how many minutes after the car reached city B would the motorcycle reach city A?

26. Given four prime numbers a , b , c , and d , if the product of $a \times b \times c \times d$ is the sum of 55 consecutive positive integers, find the smallest possible value of $a + b + c + d$.

27. A particular brand of car tyre lasts 300 km on a front wheel or 450 km on a rear wheel. By interchanging the front and rear tyres, what is the greatest distance, in km, that can be travelled using a set of four tyres of this brand?

28. In triangle ABC, AD and AE trisect angle CAB, BD and BE trisect angle CBA. If the ratio of angle C to angle D is 1: 2, find the value of angle E in degrees.



29. The sum of 10 positive integers, not necessarily distinct, is 1001. If d is the greatest common divisor of the 10 numbers, find the maximum possible value of d .

30. How many different ways are there to select 2 distinct integers from $\{2000, 2001, 2002, \dots, 2014, 2015\}$ such that the product of the 2 numbers is divisible by 6?

(Note: order is not important, choosing 2001 and 2002 is the same as choosing 2002 and 2001.)