

**SMOPS 2007 Round 1**

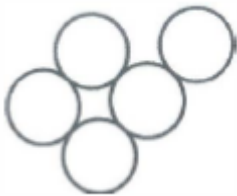
Time Duration: 2 hours

Name: \_\_\_\_\_

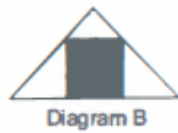
Marks: \_\_\_\_\_

1. Find the value of  $\left(1 + \frac{2}{1}\right)\left(1 + \frac{2}{2}\right)\left(1 + \frac{2}{3}\right) \times \dots \times \left(1 + \frac{2}{26}\right)\left(1 + \frac{2}{27}\right)$

2. The diagram shows 5 identical circles. On the answer sheet provided, draw a straight line to divide the figure into two parts of equal area.

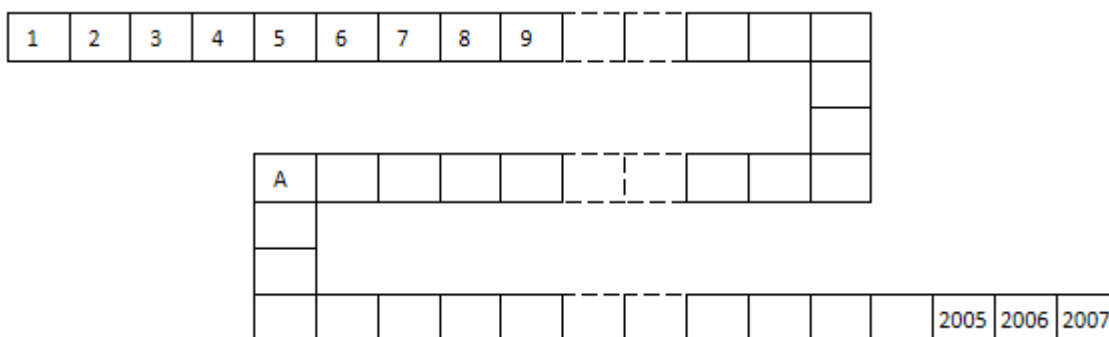


3. The diagram shows two identical isosceles right-angled triangles. If the area of the shaded square in diagram A is  $50 \text{ cm}^2$ , what is the area of the shaded in diagram B?

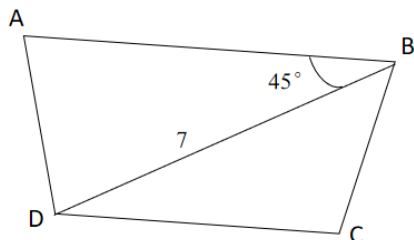


4. A rectangular wooden block measuring 30 cm by 10 cm by 6 cm is cut into as many cubes of side 5 cm as possible. Find the volume of the remaining wood.

5. The diagram shows 2007 identical rectangles arranged as shown. What number does A represent?



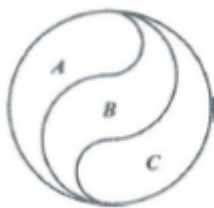
6. The diagram shows a trapezium ABCD with  $AD = BC$ . If  $BD = 7$  cm, angle  $ABD = 45^\circ$ , find the area of the trapezium.



7. An organism reproduces by simple division into two. Each division takes 5 minutes to complete. When such an organism is placed in a container, the container is filled with organisms in 1 hour. How long would it take for the container to be filled if we start with two such organism?

8. Given that  $a$ ,  $b$  and  $c$  are different whole numbers from 1 to 9, find the largest possible value of  $\frac{a+b+c}{a \times b \times c}$ .

9. The diagram comprises a circle of radius 3 cm, two semi-circles of radii 2 cm and two semi-circles of radii 1 cm. Find the ratio of the areas of the regions A, B and C.



10. In 2005, both John and Mary have the same amount of pocket money per month. In 2006, John had an increase of 10% and Mary a decrease of 10% in their pocket money. In 2007, John had a decrease of 10% and Mary an increase of 10% in their pocket money.

Which one of the following statements is correct?

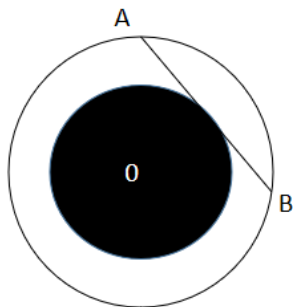
- (A) Both have the same amount of pocket money now.
- (B) John has more pocket money now.
- (C) Mary has more pocket money now.
- (D) It is impossible to tell who has more pocket money now.

11. A set of 9-digit numbers each of which is formed by using each of the digits 1 to 9 once and only once. How many of these numbers are prime?

12. Water expands 10% when it freezes to ice. Find the depth of water to which a rectangular container of base 22 cm by 33 cm and height 44 cm is to be filled so that when the water freezes completely to ice it will fill the container exactly.

13. The diagram shows two circles with centre O. Given that line AB is a chord 14 cm long and just touches the circumference of the shaded circle, find the area of the **non-shaded** region.

Take  $\pi$  as  $\frac{22}{7}$ .



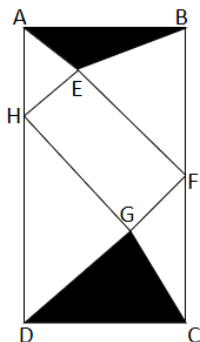
14. Joan could cycle 1 km in 4 minutes with the wind and returned in 5 minutes against the wind. How long would it take her to cycle 1 km if there is no wind? Assuming her cycling speed and the wind speed are constant throughout the journey.

15. Given that  $\sqrt{1+1 \times 2 \times 3 \times 4} = 5$ ,  $\sqrt{1+2 \times 3 \times 4 \times 5} = 11$ ,  $\sqrt{1+3 \times 4 \times 5 \times 6} = 19$  and  $\sqrt{1+4 \times 5 \times 6 \times 7} = 29$ , find the value of  $\sqrt{1+204 \times 205 \times 206 \times 207}$ .

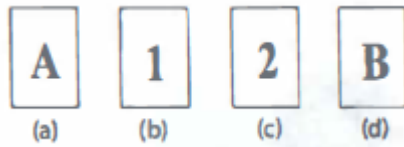
16. Peter walked once around a track and Jane ran several times around it in the same direction. They left the starting point at the same time and returned to it at the same time. In between, Jane overtook Peter twice. If she had run around the track in the opposite direction how many times would she have passed Peter? Assume that their speeds had been constant throughout the journey.

17. Three clocks, with their hour hands missing, have minute hands which run faster than normal. Clocks A, B and C each gains 2, 6 and 15 minutes per hour respectively. They start at noon with all three minute hands pointing to 12. How many hours have passed before all three minute hands next point at the same time?

18. ABCD is a rectangle and AEF, BEH, HGC and FGD are straight lines. Given that the area of the 4-sided figure EFGH is  $82 \text{ cm}^2$ , find the total area of the shaded regions.

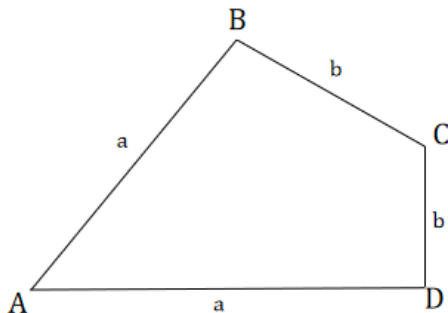


19. Four cards, **each with a letter on one side and a number on the other side**, are laid on a table.



John claims that any card with a letter **A** on one side always has the number “**1**” on the other side. Which **two** of the four cards would you turn over to check his statement?

20. The diagram shows a 4-sided figure ABCD with  $AB = AD = a$  cm and  $BC = CD = b$  cm, where  $a$  is greater than  $b$  and both  $a$  and  $b$  are whole numbers. Given that the area is  $385 \text{ cm}^2$ , find the smallest perimeter of the figure.



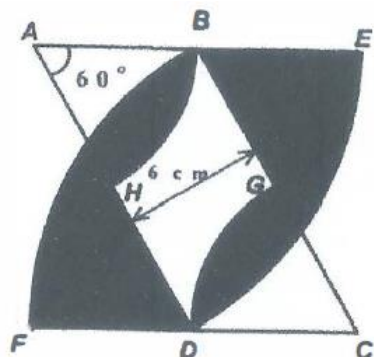
21. Find the number of ways to put 4 different coloured marbles into 4 identical empty boxes.

22. Abel and Bernard started travelling at the same instant from P and reached S at the same time. During the journey, Abel spent one third of Bernard's travelling time resting while Bernard spent one quarter of Abel's travelling time resting. Find the ratio of Abel's speed to Bernard speed.  
[Note: travelling time excludes time for resting.]

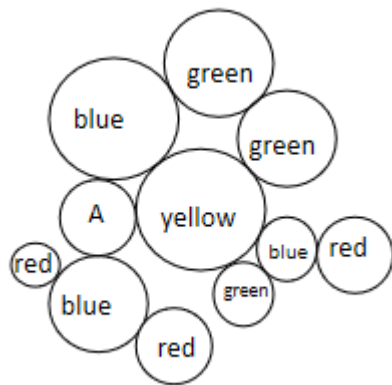
23. A family consists of 1 grandfather, 2 fathers, 1 father-in-law, 1 brother, 2 sons, 1 grandmother, 2 mothers, 1 mother-in-law, 1 daughter-in-law, 2 sisters, 2 daughters, 4 children and 3 grandchildren. What is the smallest possible number of persons in this family?

24. There are 5 schools between school A and school B. The seven schools are whole number of kilometers from each other along a straight line. The schools are spaced in such a way that if one knows the distance a person has traveled between any two school he can identify the two schools. What is the shortest distance between A and B?

25. The diagram shows a parallelogram ABCD with angle  $BAD = 60^\circ$ ,  $AB = 7$  cm,  $AD = 14$  cm and a height of 6 cm. Arcs BH and ED have centers at A and arcs BF and GD have centers at C. Given that ABE, FDC, AHD and BGC are straight lines, find the total area of the shaded regions. Take  $\pi$  as  $\frac{22}{7}$ .



26. What is the colour of circle A?



27. Find the value of  $\left(1 - \frac{1}{2^2}\right)\left(1 - \frac{1}{3^2}\right)\left(1 - \frac{1}{4^2}\right)\left(1 - \frac{1}{5^2}\right) \dots \left(1 - \frac{1}{2006^2}\right)\left(1 - \frac{1}{2007^2}\right)$

28. Three circles of radii 7 cm are enclosed by a belt as shown in the diagram below. By rearranging the circles it is possible to find the shortest possible length of the belt needed to enclose the three circles. Find this shortest length. Take  $\pi$  as  $\frac{22}{7}$ .



29. Given that  $\frac{1}{n_1} > \frac{2}{n_2} > \frac{3}{n_3} > \dots > \frac{99}{n_{99}} > \frac{100}{n_{100}}$  and  $n_1, n_2, n_3, \dots, n_{99}, n_{100}$  are different whole numbers, find the smallest value of the sum  $n_1 + n_2 + n_3 + \dots + n_{99} + n_{100}$ .

30. A ball was dropped from a height of 270 m. On each rebound, it rose to 10% of the previous height. Find the total vertical distance travelled by the ball before coming to rest.