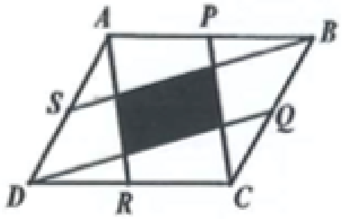
APMOPS 2012 Round 1

1. Find the value of 29999 + 2999 + 299 + 29 + 9

2. *ABCD* is a parallelogram *P*, *Q*, *R* and *S* are the midpoints of the 4 sides of the parallelogram. If the area of the shaded region is 20 cm^2, find the area of the parallelogram *ABCD*.



3. Jane added up all the digits of the whole number

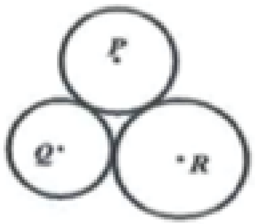
$$\underbrace{3 \times 3 \times 3 \times \ldots \times 3}_{\text{product of } 2012 \text{ of } 3's}$$

and obtained a new number n_1. She then added up all the digits of n_1 and obtained another number n_2. She continued doing this until she obtained a single digit number. Find the value of this number.

4. The diagram shows 3 circles. The circumference of the smallest circle passes through the centre of the middle circle and the circumference of the middle circle passes through the centre of the largest circle. Find the ratio of the shaded area to the unshaded area.

5. The product of 4 consecutive whole numbers is 5040. Find the value of the smallest number.

6. The diagram shows 3 circles with centres P, Q and R respectively. Each circle has a point of contact with the other circles. If $PQ = 35$ cm, $QR = 36$ cm and $PR = 37$ cm, find the radius of the circle with the centre R.

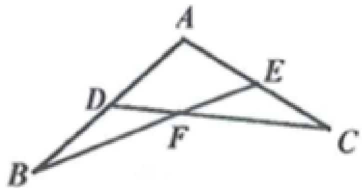


7. 12! is equal to
(1) 479001600 (2) 479000610 (3) 479000160
(4) 479000061 (5) 479000016
[Note: $n! = (n) \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$,
for example $5! = 5 \times 4 \times 3 \times 2 \times 1$.]

8. Only 1 of the 3 boys Abel, Ben and Cain can swim. Abel says, “I can swim.” Ben says, “I cannot swim.” Cain says, “Abel cannot swim.” Only 1 boy is telling the truth. Who can swim?

9. 10 boys received their test papers. The test paper has a maximum score of 10. Each boy added the scores of the other 9 boys. If the 10 totals obtained are 66, 66, 67, 67,68,68,69,70,71 and 72, find the lowest score.

10. In the diagram, D and E are the mid-points of AB and AC respectively. BE cuts CD at F. If the area of the 4-sided figure $ADFE$ is 256 cm^2, find the area of the triangle ABE.



11. A circle with centre O passes through points A and B as shown. If the circle has a radius of 5 cm and angle AOB is 120°, find the radius of another circle that passes through points O, A and B.

12. A line can divide a plane into a maximum of 2 regions. 2 lines can divide a plane into a maximum of 4 regions. Find the number of regions that 4 lines can divide a plane into.

13. Find the value of
$$\frac{1}{1}+\frac{1}{2}+\frac{2}{2}+\frac{1}{3}+\frac{2}{3}+\frac{3}{3}+\frac{1}{4}+\frac{2}{4}+\frac{3}{4}+\frac{4}{4}+\cdots+\frac{1}{100}+\frac{2}{100}+\frac{3}{100}+\cdots+\frac{98}{100}+\frac{99}{100}+\frac{100}{100}$$

14. Trains A and B are travelling towards each other at 48 km/h. In train A, Jane notices that it takes 6 seconds for train B to pass her. Find the length of train B in m.

15. Find the largest while number smaller than

$$\frac{1}{\frac{1}{101}+\frac{1}{102}+\cdots+\frac{1}{109}+\frac{1}{110}}$$

16. Find the largest whole number k such that

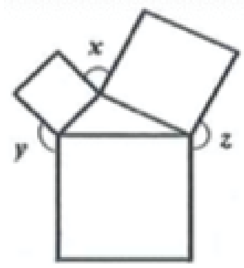
$$\underbrace{12\times 12\times 12\times \ldots \times 12}_{50} > \underbrace{k\times k\times k\times \ldots \times k}_{75}$$

17. Jane was given a sum of money for her 10 day trip. At the end of day 1, she spent $\frac{1}{10}$ of the money. At the end of day 2, she spent $\frac{1}{9}$ of the remaining sum. At the end of day 3, she spent $\frac{1}{8}$ of the remaining sum … At the end of day 9, she spent $\frac{1}{2}$ of the remaining sum and has \$99 left. How much money did she have at the beginning?

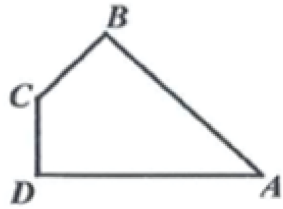
18. 4 classes *A*, *B*, *C* and *D* each has less than 50 students and the average number of students is 46. Class *A* and class *B* differ in number of students by 4. Class *B* and class *C* by 3, and class *C* and class *D* by 2. If class *A* has the most number of students, find the number of students in *A*.

19. The diagram shows a figure comprising of 4 regions. 4 different colours are used at most to colour the figure such that each region is coloured by only 1 colour and regions adjacent to each other cannot have the same colour. Find the number of different ways to colour the figure.

20. The figure consists of 3 squares and a triangle. Find the sum of the angles x, y and z in degrees.

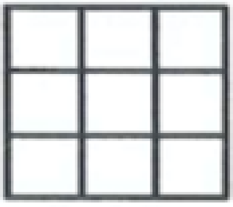


21. In the diagram, AD = 6 cm, BC = 2 cm, angle ABC = angle ADC = 90° and angle BCD = 135°. Find the area of the 4-sided figure ABCD in cm^2.



22. A bus was scheduled to travel from Town A to Town B at a constant speed of x km/h. If the speed of the bus increased by 20%, it could arrive at Town B 2 hours ahead of schedule. If the bus travelled the first 240 km at x km/h, and then the speed decreased to 80%, it could arrive at Town B 2 hours behind schedule. Find the distance, in km, between the 2 towns.

23. In how many ways can we shade exactly 2 of the 9 squares such that the 2 shaded squares have no sides in common?



24. Jane has an alarm clock that is slower by 5 minutes for every actual hour. One night, Jane resets the clock correctly at 2100. If she wanted the alarm clock to ring at 0700 (the actual time) the following morning, what is the time she should set for the clock to ring? [2100 denotes 9pm, 0700 denotes 7am]

25. A particular month has 5 Tuesdays. The first and the last day of the month are not Tuesdays. The last day of the month is a __________.

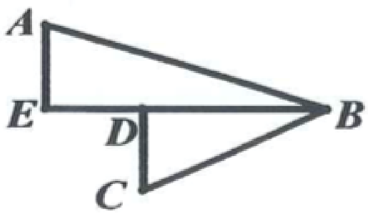
26. Find the value of

$$\left(1-\frac{1}{2}-\frac{1}{3}-\cdots-\frac{1}{2011}\right)\times\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{2011}+\frac{1}{2012}\right)-\left(1-\frac{1}{2}-\frac{1}{3}-\cdots-\frac{1}{2011}-\frac{1}{2012}\right)\times\left(\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{2011}\right)$$

27. A cross shaped figure comprising 5 identical squares is cut into 3 pieces A, B and C by 2 straight cuts as indicated by the dotted lines. A, B and C can be rearranged to form a rectangle as shown. If the length of the rectangle is 12 cm, find the width of the rectangle in cm.

28. David wants to go from level 2 to level 1 in a shopping centre. If he walks down 14 steps while taking the escalator, he can move from the top to the bottom of the escalator in 30 seconds. If he walks down 28 steps, he can do the same in 20 seconds. Find the number of steps of the escalator.

29. ABE and BCD are right-angles triangles. D lies on BE such that $AE = ED = DC = 1$ cm and $DB = 2$ cm. Find the value of angle ABC in degrees.



30. A 6-digit number $\overline{abcdef}$ is such that $\overline{defabc} = 6 \times \overline{abcdef}$. Find the 6-digit number $\overline{abcdef}$.