

2023 RIPMWC Mock Exam Solution

1. Calculate: $\frac{\frac{1}{2}}{1+\frac{1}{2}} + \frac{\frac{1}{3}}{(1+\frac{1}{2}) \times (1+\frac{1}{3})} + \dots + \frac{\frac{1}{2023}}{(1+\frac{1}{2}) \times (1+\frac{1}{3}) \times \dots \times (1+\frac{1}{2023})}$

A. $\frac{1011}{1012}$

B. $\frac{2022}{2023}$

C. $\frac{2021}{2022}$

D. $\frac{1010}{1011}$

E. None of the above

【Answer】 A

【Solution】

Find the general term: $\frac{\frac{1}{n+1}}{(1+\frac{1}{2}) \times (1+\frac{1}{3}) \times \dots \times (1+\frac{1}{n+1})} = \frac{\frac{n+1}{n+2}}{\frac{n+1}{2}} = \frac{2}{(n+1)(n+2)} = 2 \times (\frac{1}{n+1} - \frac{1}{n+2})$

The original question can be simplified into $\left[(\frac{1}{2} - \frac{1}{3}) + (\frac{1}{3} - \frac{1}{4}) + (\frac{1}{4} - \frac{1}{5}) + \dots + (\frac{1}{2023} - \frac{1}{2024}) \right] \times 2$

which is equal to $\left[(\frac{1}{2} - \frac{1}{2024}) \right] \times 2 = \frac{1011}{1012}$

2. The table below contains 100 numbers. What is their sum?

2011	2012	2013	...	2019	2020
2012	2013	2014	...	2020	2021
2013	2014	2015	...	2021	2022
⋮	⋮	⋮	⋮	⋮	⋮
2020	2021	2022	...	2028	2029

A. 201550

B. 202300

C. 202000

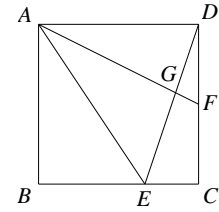
D. 201620

E. None of the above

【Answer】 C

【Solution】 The numbers on the diagonal connecting top right and bottom left are all 2020. When folded along this diagonal, the sum of the two numbers overlapping is 4040. So, the average of these 100 numbers is 2020, the sum of these 100 numbers = $2020 \times 100 = 202000$.

3. In the figure below, there is a square $ABCD$ with side length of 1. $BE = 2EC$, $CF = FD$, find the area of triangle AEG .



- A. $\frac{1}{3}$
- B. $\frac{2}{5}$
- C. $\frac{3}{7}$
- D. $\frac{2}{7}$
- E. None of the above

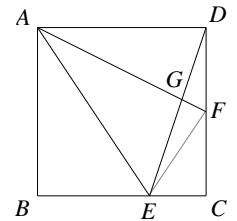
【Answer】 D

【Solution】 Connect EF. Since $BE = 2EC$, $CF = FD$, $S_{\triangle DEF} = (\frac{1}{2} \times \frac{1}{3} \times \frac{1}{2})S_{\square ABCD} = \frac{1}{12}S_{\square ABCD}$.

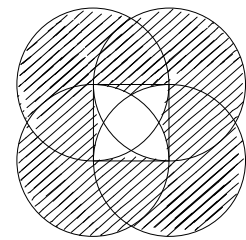
Because $S_{\triangle AED} = \frac{1}{2}S_{\square ABCD}$, using the butterfly model, $AG : GF = \frac{1}{2} : \frac{1}{12} = 6 : 1$,

Therefore $S_{\triangle AGD} = 6S_{\triangle GDF} = \frac{6}{7}S_{\triangle ADF} = \frac{6}{7} \times \frac{1}{4}S_{\square ABCD} = \frac{3}{14}S_{\square ABCD}$.

So, $S_{\triangle AGE} = S_{\triangle AED} - S_{\triangle AGD} = \frac{1}{2}S_{\square ABCD} - \frac{3}{14}S_{\square ABCD} = \frac{2}{7}S_{\square ABCD} = \frac{2}{7}$.



4. As shown in the figure below, the square in the middle has a side length of 1. The four vertices of the square are the centers of four circles, and the four sides of the square are the radii of these four circles. Find the area of the entire shaded region. (take π as 3.14)



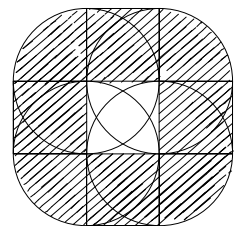
- A. 8.28
- B. 7.14
- C. 7.28
- D. 8.14
- E. None of the above

【Answer】 B

【Solution】

Move the four small shadows inside the middle square outwards to obtain the figure as shown on the right.

$$S_{\text{shaded area}} = 4 \times S_{\text{square}} + 4 \times S_{\frac{1}{4}\text{round}} = 4 \times S_{\text{square}} + S_{\text{round}} = 4 \times 1^2 + \pi \times 1^2 = 4 + \pi = 7.14$$



5. Starting from the number “2023”, Alex writes down a string of numbers such that each number is the unit digit of the product of its previous two numbers. For example, $2 \times 3 = 6$, so Alex writes down “6”, $3 \times 6 = 18$, so Alex writes down “8” after “6”... In the end, Alex obtained a string of numbers: 20236884... Counting from the leftmost number which is the number “2”, what is the 2023rd digit of this string of numbers?

- A. 3
- B. 6
- C. 8
- D. 4
- E. None of the above

【Answer】 C

【Solution】 The first few numbers are as follows: 2023688428688428 with "688428" repeating. Since $(2023 - 4) \div 6 = 336R3$, the 2023rd number is 8.

6. There are 4 identical bottles, each containing a certain amount of oil. By putting two bottles together on the scale every time, the weights are recorded as follows: 8, 9, 10, 11, 12, 13. If the total weight of the 4 bottles when they are empty is a prime number, the total weight of the oil from the 4 bottles (not including the weight of the bottles) is also a prime number, what is the total weight of the oil from the two heaviest bottle (not including the weight of the bottles).

- A. 11
- B. 12
- C. 11.5
- D. 10.5
- E. None of the above

【Answer】 B

【Solution】 Since each bottle was weighed three times, the sum of the recorded data is triple the sum of the weights for the 4 bottles of oil (with bottles), it follows that the 4 bottles of oil (with bottles) weigh a total of $(8 + 9 + 10 + 11 + 12 + 13) \div 3 = 21$ (kg). Since both the sum of the oil weights and the sum of the bottle weights are prime numbers, so they must be one odd and one even. Since 2 is the only even prime number, there are only two possibilities: (1) the sum of the oil weights is 19 kg, the sum of the bottle weights is 2 kg, each bottle weighs $\frac{1}{2}$ kg, and the two heaviest bottles contain oil is $13 - \frac{1}{2} \times 2 = 12$ (kg). (2) The sum of the oil weights is 2 kg, the sum of the bottle weights is 19 kg and each bottle weighs $\frac{19}{4}$ kg, the heaviest two bottles contain $13 - \frac{19}{4} \times 2 = \frac{7}{2}$ (kg) of oil, which contradicts the sum of the oil weights of 2 kg. Therefore, the two heaviest bottles contain a total of 12 kg of oil.

7. Two cars, X and Y, both depart from place A and keep travelling back and forth between place A and place B nonstop: once they reach place B, they immediately turn back and travel towards place A; once they reach place A, they immediately turn back and travel towards place B. The two cars meet each other for the first and second time both at place C which is somewhere between place A and B. Given that the speed of car X is faster than car Y, the speed of car X is how many times the speed of car Y?

- A. 2
- B. 3
- C. 1.5
- D. 2.5
- E. None of the above

【Answer】 A

【Solution】 At the first encounter, the two cars travelled two full distances together, while car Y travelled AC. After the first encounter, the two cars travelled another two full distances together at the second encounter while car B travelled from C to B and back again, i.e. 2BC. Since the total distance travelled is the same in both cases, the distance travelled by car Y is also the same in each case, so the length of AC is equal to 2 times the length of BC. The speed of car X and car Y is 2:1, so the speed of car X is twice the speed of car Y.

8. In a community club, the ratio of male members to female members is 3:2. The members are put into three groups A, B and C where the ratio of the number of members is 10:8:7. In group A, the ratio of male to female is 3:1, in group B, the ratio of male to female is 5:3. What is the ratio of male to female in group C?

- A. 4:9
- B. 1:2
- C. 2:5
- D. 5:7
- E. None of the above

【Answer】 E

【Solution】 Let the total number of members be 1, the number of male members in group A is $\frac{10}{10+8+7} \times \frac{3}{3+1} = \frac{3}{10}$ and female member in group A is $\frac{3}{10} \times \frac{1}{3} = \frac{1}{10}$, the number of male members in group B is $\frac{8}{10+8+7} \times \frac{5}{5+3} = \frac{1}{5}$ and female members in group B is $\frac{1}{5} \times \frac{3}{5} = \frac{3}{25}$, and the number of male members in group C is $\frac{3}{3+2} - \left(\frac{3}{10} + \frac{1}{5} \right) = \frac{1}{10}$ and female members in group C is $\frac{2}{3+2} - \left(\frac{1}{10} + \frac{3}{25} \right) = \frac{9}{50}$; therefore, the ratio of the number of male and female members in group C is $\frac{1}{10} : \frac{9}{50} = 5:9$.

9. Solution A, B and C are 48% alcohol solution, 62.5% alcohol solution and $\frac{2}{3}$ alcohol solution respectively. The total weight of the three solutions is 100kg and the weight of solution A is equal to the weight of solution B and C combined. If all three solutions are mixed, the result is a 56% alcohol solution. How many kg of alcohol are there in solution C originally?

- A. 12
- B. 15
- C. 17
- D. 21
- E. None of the above

【Answer】 A

【Solution】 Let the weight of solution C be x ,

$$50 \times 48\% + (50 - x) \times 62.5\% + x \times \frac{2}{3} = 100 \times 56\%$$

So, $x = 18$, the weight of pure alcohol in solution C is $18 \times \frac{2}{3} = 12$ (kg).

10. A 6-digit number N consists of six distinct non-zero numbers and N is divisible by 11. By rearranging the six digits of N , at least how many new 6-digit numbers (not including the number N) can be obtained such that they are also divisible by 11?

- A. 36
- B. 35
- C. 72
- D. 71
- E. None of the above

【Answer】 D

【Solution】 Given that this six-digit number is \overline{abcdef} , then the difference between $(a + c + e)$ with $(b + d + f)$ is either 0 or a multiple of 11. Consider first the in-group exchange within the even digits of a, c, e and within the odd digits of b, d, f . There are $P_3^3 \times P_3^3 = 36$ ways of rearranging. Then, consider the case like \overline{badcfe} between odd and even digits, which also has $P_3^3 \times P_3^3 = 36$ orders. Therefore, there are at least $36 + 36 = 72$ numbers (including the original \overline{abcdef}) that are divisible by 11. So, at least $72 - 1 = 71$ six-digit numbers divisible by 11.

11. Three classes A, B and C from Guang Ming Primary School were asked to put up 14 shows during their School Annual Day. If every class must put up at least 3 shows, how many possible ways can the 14 shows be assigned to the three classes?

- A. 42
- B. 21
- C. 5
- D. 30
- E. None of above

【Answer】 B

【Solution】 Partitioning method. Assign each class 2 shows first such that there are 8 shows left. Using the partitioning method, there are $7 \times 6 \div 2 = 21$ possible ways.

12. Out of all the natural numbers that are strictly less than 2023 (including 0), how many numbers are there that give the same remainder when divided by 20 and when divided by 23? (Remainder can be zero).

- A. 4
- B. 5
- C. 99
- D. 100
- E. None of above

【Answer】 D

【Solution】 We know that the least common multiple of 20, 23 is 460, so it will appear once every 460 numbers. Between 1 and 460, only 1, 2, 3, ..., 19, 460 these 20 numbers when divided by 20 and 23 give the same remainder. Since $2023 \div 460 = 4R183$, there are $4 \times 20 + 19 = 99$ such numbers. But "0" is also divided by both 20 and 23, so there are 100 such numbers.

13. There are 10 multiple choice questions in a math contest. Each participant gets 10 points at the beginning. For every question answered correctly, 3 points will be added. For every question answered incorrectly, 1 point will be deducted. For every unattempt question, no points will be added or deducted. At least how many participants is needed to guarantee 4 participants get the same marks?

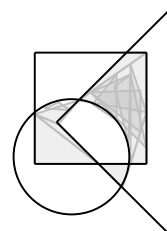
- A. 124
- B. 121
- C. 115
- D. 114
- E. None of above

【Answer】 C

【Solution】 The possible marks range from 0 to 40 points. But it is impossible to get 39, 38 and 35. So, to ensure that at least 4 people score the same, at least $3 \times (41 - 3) + 1 = 115$ participants are needed.

14. In the figure below, a triangle, a square and a circle are placed on top of each other. Given that they each has area 60cm^2 , the total area of the shaded region is 40cm^2 and the total area covered by the three shapes is 100cm^2 , what is the area of the overlapping part of all three shapes?

- A. 40
- B. 35
- C. 30
- D. 20
- E. None of above



【Answer】 D

【Solution】 The shaded part is double counted while the three overlapping pieces of paper are counted three times. So, the area of the overlapping part of the three sheets of paper $= (60 \times 3 - 100 - 40) \div 2 = 20 (\text{cm}^2)$.

15. When Lilia started writing her homework sometime between 8am and 9am in the morning, the hour hand and minute hand overlapped each other exactly. When Lilia finished writing her homework sometime between 10am and 11am, the hour hand and minute hand overlapped each other exactly again. How long did it take for Lilia to finish writing her homework?

- A. 2h10min
- B. 2h11min
- C. $2\text{h}10\frac{10}{11}\text{min}$
- D. $2\text{h}10\frac{7}{11}\text{min}$
- E. None of above

【Answer】 C

【Solution】 Between 8 and 9 o'clock, the hour and minute hands coincide at:

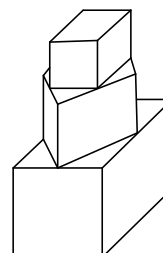
$40 \div \left(1 - \frac{1}{12}\right) = 43\frac{7}{11}$ (minutes). Between 10 and 11 o'clock, the hour and minute hands coincide

at: $50 \div \left(1 - \frac{1}{12}\right) = 54\frac{6}{11}$ (minutes).

So, $10\text{hours}54\frac{6}{11}\text{minutes} - 8\text{hours}43\frac{7}{11}\text{minutes} = 2\text{hours}10\frac{10}{11}\text{minutes}$, Lilia took

$2\text{hours}10\frac{10}{11}\text{minutes}$ to finish her homework.

16. Lawrence stacked some cubes to form a tower such that the four bottom corners of every cube must touch the midpoint of the edges of the cube that it is stacked on (the figure below is an example of this with three cubes). Given that the edge length of the lowest cube is 2 and the total surface area of the tower exceeds 39 (including the bottom area of the lowest cube), at least how many cubes are there in the tower Lawrence built?



- A. 5
- B. 6
- C. 7
- D. 8
- E. None of the above

【Answer】 B

【Solution】 The tower has a fixed upper and lower surface area, no matter how many layers it has, $2 \times 2 + 2 \times 2 = 8$. The area of each of its sides should be more than $(39 - 8) \div 4 = 7.75$. The area of a single side of the lowest square is $2 \times 2 = 4$, and those on top follows the pattern $2, 1, \frac{1}{2}, \frac{1}{4}, \dots$. The sum of the areas of one side of the first five cubes is $4 + 2 + 1 + \frac{1}{2} + \frac{1}{4} = 7.75$. So, to exceed 7.75, we need at least 6 cubes.

17. $\lfloor a \rfloor$ denotes the greatest integer less than or equal to a . For example, $\lfloor 1.9 \rfloor = 1$

and $\left\lfloor \frac{17}{4} \right\rfloor = 4$. Find the value of $\left\lfloor \frac{1}{\frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \dots + \frac{1}{19}} \right\rfloor$

- A. 0
- B. 1
- C. 2
- D. 3
- E. None of the above

【Answer】 B

【Solution】 $\frac{1}{\frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \dots + \frac{1}{19}} > \frac{1}{\frac{1}{10} + \frac{1}{10} + \frac{1}{10} + \dots + \frac{1}{10}} = \frac{1}{\frac{10}{10}} = 1$
 $\frac{1}{\frac{1}{10} + \frac{1}{11} + \frac{1}{12} + \dots + \frac{1}{19}} < \frac{1}{\frac{1}{19} + \frac{1}{19} + \frac{1}{19} + \dots + \frac{1}{19}} = \frac{1}{\frac{10}{19}} = 1.9$,

18. Among the 2023 natural numbers from 1 to 2023, how many number a are there such that $2023 + a$ is divisible by $2023 - a$.

- A. 11
- B. 10
- C. 12
- D. 13
- E. None of the above

【Answer】 B

【Solution】

If $\frac{2023+a}{2023-a}$ is an integer, it follows that $\frac{2023+a}{2023-a} + 1 = \frac{2023+a+2023-a}{2023-a} = \frac{4046}{2023-a}$ is also an integer. This means that $2023-a$ is a factor of 4046, we 4046 has 12 factors, of which 4046 is the largest one. But obviously there is no value for a such that $2023-a$ equals 4046. On top of that, since a can't be 0. Therefore, the remaining 10 values can all have corresponding values of a , so there are 10 values for a that satisfy the condition.

19. Worker A, B and C working alone can finish a project in 36, 30 and 48 days respectively. If worker A, B and C work together on the same project but worker C rests for X days (X is a whole number) while worker A and B never rest, the entire project can be finished in Y days (Y is a whole number). What is the value for X ?

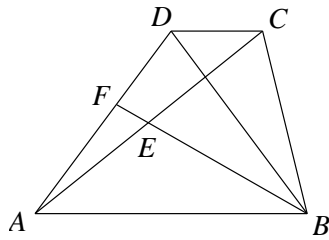
- A. 9
- B. 10
- C. 11
- D. 12
- E. None of the above

【Answer】 C

【Solution】 Assume we need a days to complete the project, during which C rested b days.

$\left(\frac{1}{36} + \frac{1}{30} + \frac{1}{48}\right)a - \frac{1}{48}b = 1, \frac{59}{720}a - \frac{1}{48}b = 1, 59a - 15b = 720$. From the equation above, since $15b$ and 720 are both multiples of 15, $59a$ must be a multiple of 15, so a is a multiple of 15. Under the condition that $a > b$, only $a = 15$ and $b = 11$ are possible, which means that C rested for 11 days.

20. As shown in the figure below, the trapezium $ABCD$ has an area of 12. If $AB=2CD$, E is the midpoint of AC and BE is extended to F such that BF and AD intersect at point F . The area of quadrilateral $CDEF$ is _____.



- A. 2
 B. $\frac{7}{3}$
 C. $\frac{7}{4}$
 D. $\frac{8}{3}$
 E. None of the above

【Answer】 D

【Solution】 Extend BF and CD to intersect at G .

Since E is the midpoint of AC , according to the similar triangle property, $CG = AB = 2CD$

and $GD = \frac{1}{2}GC = \frac{1}{2}AB$. Then according to the similar triangle property,

$$AF : FD = AB : DG = 2 : 1 \quad \text{and} \quad GF : GB = 1 : 3, \quad \text{and} \quad S_{\triangle ABD} : S_{\triangle BCD} = AB : CD = 2 : 1.$$

$$\text{So } S_{\triangle BCD} = \frac{1}{3}S_{\triangle ABCD} = \frac{1}{3} \times 12 = 4 \quad \text{and} \quad S_{\triangle GBC} = 2S_{\triangle BCD} = 8.$$

$$\text{And } \frac{S_{\triangle GDF}}{S_{\triangle GBC}} = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}, \quad S_{\triangle EBC} = \frac{1}{2}S_{\triangle GBC}, \quad \text{so } S_{CDEF} = \left(1 - \frac{1}{2} - \frac{1}{6}\right)S_{\triangle GBC} = \frac{1}{3}S_{\triangle GBC} = \frac{8}{3}.$$

