

APMOPS 2015 Round 2

Time Duration: 2 hours

Name: _____

Marks: _____

1. A magician invited an audience to write a secret six-digit number N and hid it from his view. Let the sum of digits of N be S .

The audience revealed the digits in $N - S$, in a random order: 0, 2, 4, 6 and 8.

The magician correctly predicted the final 6th digit.
What digit is it? Explain your reasoning.

*Bonus question: Find the minimum and maximum value of N .

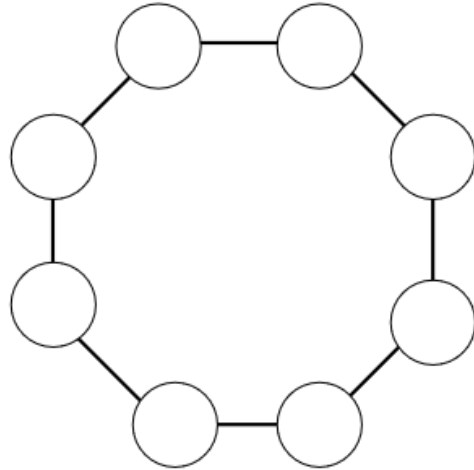
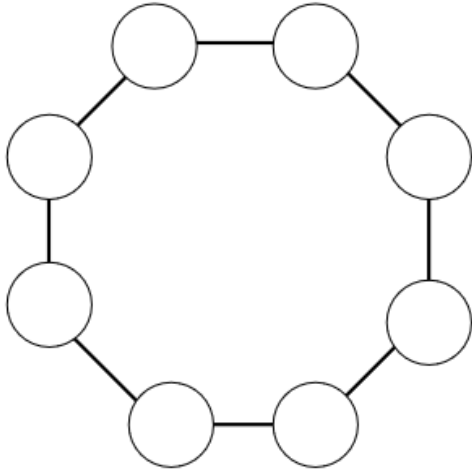
2. In the figure below, the eight vertices of an octagon are assigned digits from 1 to 8, where each digit is used exactly once.

Can the sum of every four consecutive circles be:

I) larger than 16?

II) larger than 17?

If possible, find a way of doing so; if not, explain your reasoning.



3. Mr Wong cycled at a constant speed from bus terminal A to bus terminal B .

Buses from A and B leave the terminals at the same time interval (e.g. if a bus leaves terminal A every thirty minutes, so will terminal B), but at different times.

Every 6 minutes, Mr Wong met a bus coming from the opposite direction, and every 9 minutes he was overtaken by a bus travelling in the same direction as him.

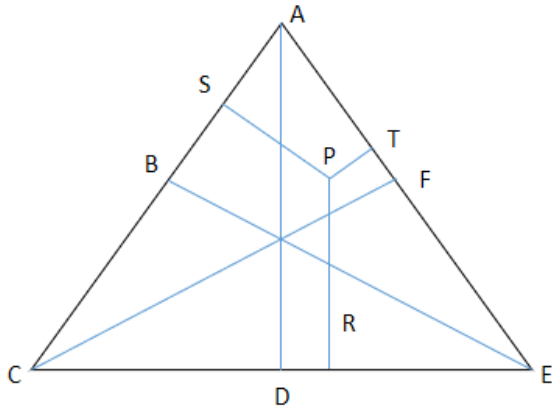
Given all buses take 50 minutes to travel from A to B and vice versa, how long did it take for Mr Wong to travel from A to B ?

4. In the figure below, triangle ACE has altitudes (perpendicular heights) AD , BE and CF .

P is a point in the triangle such that $PR \parallel AD$, $SP \parallel BE$ and $PT \parallel CF$.

Given $AD = 2010$, $BE = 2013$, $CF = 2016$; $PR = 1005$, $PS = 671$.

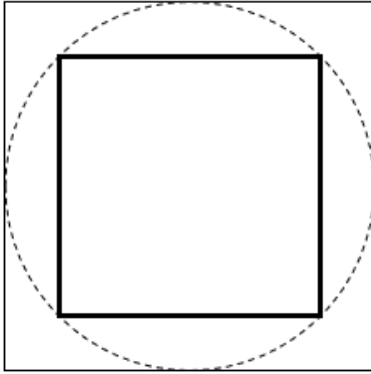
Find the length of PT .



5. As shown in the figure below, a square of area 1 cm^2 is enclosed by a circle touching its four corners. A new square is then drawn touching four points on the circle.

This process is then repeated a number of times.

Find the least number of times this process must be repeated, so that the area of the new square is more than 1 km^2 .



6. How many different ways are there to paint the six faces of a cube so that no two adjacent faces have the same colour?

Each face must be painted only one colour.

Five colours are available but it is not necessary to use all of them.

(Two ways of painting which give the same colour on corresponding faces after rotations or flips are considered the same way.)