APMOPS 2012 Round 2

Time Duration: 2 hours   Name: _____   Marks: _____
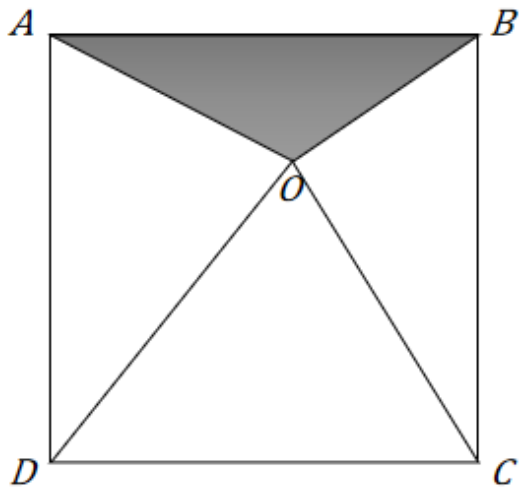
1. There is a group of girls at a party. Each girl writes a unique 2-digit number on the board. Jane claims that the sum of digits is the same for at least 3 numbers.

Find the smallest number of girls in the room for this statement to be true.

2. Abel and Ben work on a job. Abel can finish the job in 10 hours and Ben can finish the job in 12 hours. Abel works on the job for one hour, then hands over the job to Ben, who works for one hour, then hands it back to Abel.

If they continue in the aforementioned pattern, how long, in minutes, will it take to finish the entire job?

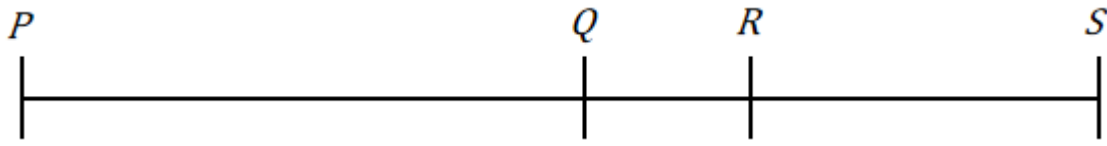3. O is a point in the square ABCD. Line BO is 4 cm and Line OC is 7 cm. Angle BOC is 90°. Find the shaded area.

4. A very large number is formed by writing consecutive natural numbers as follows:

$$12345678910111213141516...$$

Find the last two digits of this number when it is divisible by 72 for the first time.

5. The diagram below shows a line PS. The distance between each point is a whole number.

Points Q is the midpoint of line PS.
Given that

$$PQ \times PR \times PS \times QR \times QS \times RS = 10500$$

Find the length of line PS.

6. A straight line is dropped from a triangle ABC, dividing it into 2 isosceles triangles. Given that one of the interior angles of triangle ABC is 24°, find all the possible greatest interior angles of triangle ABC.