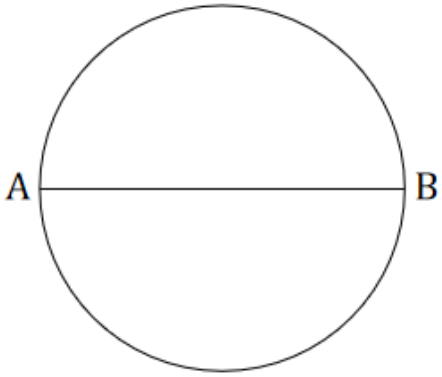
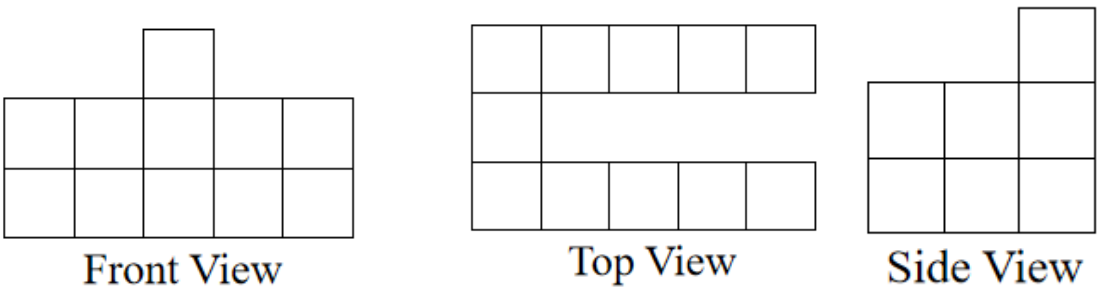
APMOPS 2018 Round 2

Time Duration: 2 hours Name: _____ Marks: _____

1. 2 people *X* and *Y* start from *A* and *B* respectively. If *X* travels clockwise, while *Y* travels counter-clockwise, they will meet after 50 seconds. If both of them travel clockwise, *X* will catch up with *Y* after 300 seconds. Find the ratio of *X*'s speed to *Y*'s speed. (*AB* is the diameter of the circle)

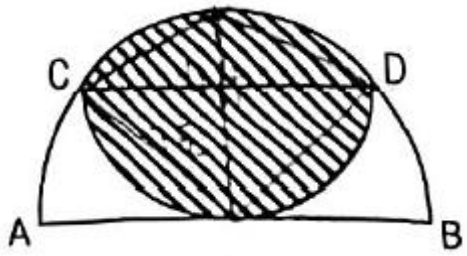


2. A figure made up of $1\times1\times1$ cubes has the following front, top and side view.



What is the smallest possible number of $1\times1\times1$ cubes needed to construct this figure?

3. If the diagram, a semicircle is inscribed in another semicircle such that *AB* is the chord of the larger semicircle while *CD* is the diameter. Given that *CD*= 14, find the shaded area. (Take the $\pi = \frac{22}{7}$)



4. A 3-digit number $\overline{abc}$ has the following property:

$\overline{abc}$ is divisible by 9.

$\overline{bc}$ is divisible by 13.

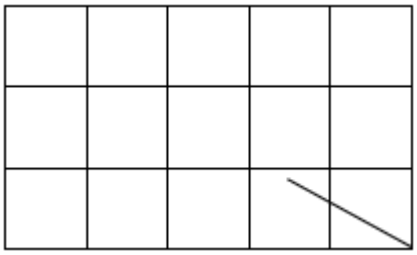
$\overline{ab}$ is a prime number.

Find all possible values of $\overline{abc}$.

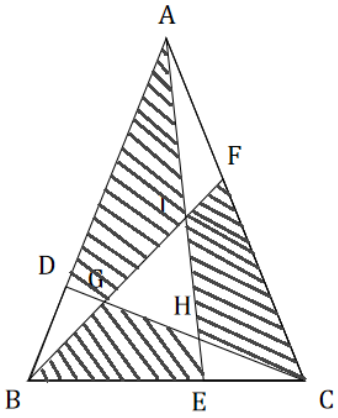
5. Put 60 identical balls into 10 identical boxes such that each box has at least one ball while all boxes contain a different number of balls. How many ways can you do so?

6. A $10\times10\times10$ cube is formed by $1\times1\times1$ dice. The sum of the numbers on opposite faces of a dice is 7. Find the smallest sum of the numbers on the surfaces of the $10\times10\times10$ cube.

7. A diagonal cut through a 1965×2018 rectangle. If the rectangle is formed by $1 \times 1 \times 1$ square, find the number of the squares the diagonal cuts through.



8. In the triangle ABC, $\frac{BD}{AB} = \frac{AF}{AC} = \frac{CE}{BC} = \frac{1}{3}$. Given that the area of the triangle $ABC = 588$ and the area of triangle $GHI = 84$, find the shaded area.



9. A 4×2 table is formed by 1×1 squares. You are given 6 colours to colour the squares such that no adjacent squares have the same colour. How many ways can you colour the table?

10. A, B, C and D represent distinct digits such that:

$$\overline{4AB}\times\overline{C4}=\overline{35DDA}$$

Find the value of $A+B+C+D$.

11. Put red markings on a rod such that the rod is equally split into 10 pieces. Put blue markings on a rod such that the rod is equally split into x pieces. If the red markings and the blue markings are put on the same rod, such that it is split into 20 pieces, determine all the values of x.

12. When a 6-digit number is divided by 100, we get a quotient and a remainder (can be 0). The sum of the quotient and the remainder is a multiple of 11. How many such numbers are there?

13. An operation is performed when all the square numbers from 1 to *n* are removed. Then, the numbers are rearranged. How many operations are needed to reduce 8102 to 1?

14. Three 3-digit numbers are formed by 1-9. Each number is used only once. The second number must be two times as much as the first number while the third number must be three times as much as the first number. Determine the number of triplets that satisfies the above conditions. (for example $192 + 384 = 576$)