2023 NMOS Mock Test Solution

1. $5.2 \div 3\frac{1}{5} - 1.5 \times \frac{5}{12} =$

   [Solution] $\frac{52}{10} \div \frac{16}{5} - \frac{3}{2} \times \frac{5}{12} = \frac{52}{10} \times \frac{5}{16} - \frac{3}{2} \times \frac{5}{12} = \frac{13}{2} \times \frac{1}{4} - \frac{1}{2} \times \frac{5}{4} = \frac{13}{8} - \frac{5}{8} = 1$

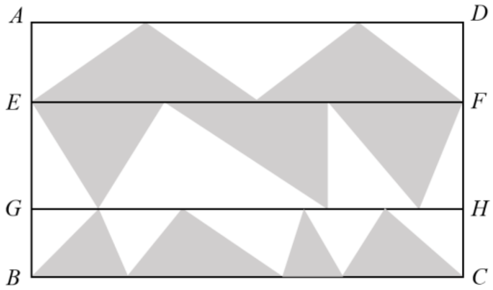
2. Given that $a \oplus b = a + b - 1$ and $a \otimes b = ab - 2$. $4 \otimes [(6 \oplus 8) \oplus (3 \otimes 5)] =$

   [Solution] $4 \otimes [(6+8-1) \oplus (3 \times 5 - 2)] = 4 \otimes [13 \oplus 13] = 4 \otimes [13 + 13 - 1] = 4 \otimes 25$
   $= 4 \times 25 - 2 = 98$

3. Alex wrote down a list of numbers: 7, 0, 2, 5, 3, 7, 0, 2, 5, 3, 7, 0, … Write is the 81st number in this list?

   [Solution] The answer is 7 because $81 \div 5 = 16R1$ and the first number in every group is 7.

4. In the figure below, quadrilateral *ADFE*, *EFHG* and *GHCB* are three rectangles in the rectangle *ADCB*. The length of *AB* is 6 cm, and the length of *BC* is 8 cm. What is the area of the shaded region in square centimeters?



   [Solution] Half model: $8 \times 6 \div 2 = 24$

5. Cyclist A and B start riding bicycles from the same point on a circular road at the same time, cycling in opposite directions. It is known that person A takes 70 minutes to complete one full lap. If person A and person B meet 45 minutes after their departure, how long does it take for person B to complete one full lap in minutes?

   [Solution] Cyclist A cycle for 45 minutes, meet cyclist B and it takes an additional 25 minutes to complete one full lap. This shows that distance traveled by A in 25 minutes is equivalent to the distance traveled by B in 45 minutes. Since it takes A 70 minutes to complete one full lap, Person B would require $70 \div 25 \times 45 = 126$ minutes to complete one full lap.

6. $2009 \times 20082007 - 2007 \times 20082009 =$

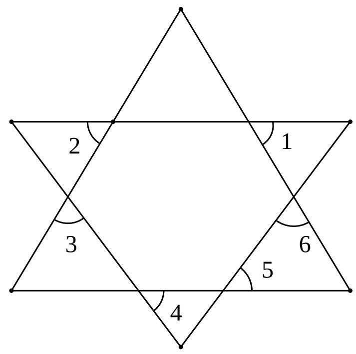
   [Solution] $2009 \times 20082007 - 2007 \times (20082007 + 2)$
   $= (2009 - 2007) \times 20082007 - 4014$

$= 40160000$

7. A boat travels 80 km downstream along a river in 2 hours at a constant speed and takes 8 hours to return upstream to the starting point. Find the speed (in km/h) of the boat in still water.

   [Solution] We have $v_{boat} + v_{current} = 40$ and $v_{boat} - v_{current} = 10$. So, $v_{boat} = 25$ km/h.

8. Find the value of $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6$ (in degree)



   [Solution] $(180° - \angle 1) + (180° - \angle 2) + (180° - \angle 3) + (180° - \angle 4) + (180° - \angle 5) + (180° - \angle 6) = 720°$

   $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 = 180° \times 6 - 720° = 360°$

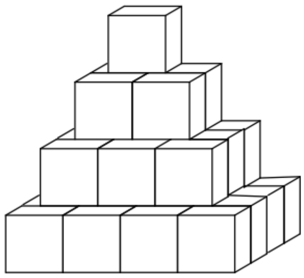
9. For a project, it takes 21 days for worker A to complete it alone, and it takes 12 days for workers A and B to complete it together. How many days would it take for worker B to complete the project alone?

   [Solution] $\frac{1}{12} - \frac{1}{21} = \frac{1}{28}$. So, it takes B 28 days to complete the project alone.

10. The length of a train is 800 meters, and its speed is 60 kilometers per hour. There are two tunnels on the railway, some distance apart. The train takes 2 minutes to pass through the first tunnel and 3 minutes to pass through the second tunnel (the time here refers to the time between the head of the train entering the tunnel and the tail leaving the tunnel). The total time taken to pass through both tunnels is 6 minutes. Find the distance between the two tunnels in meters.

    [Solution] The train's speed is 1000 meters per minute. The length of the first tunnel is $1000 \times 2 - 800 = 1200$ meters and the length of the second tunnel is $1000 \times 3 - 800 = 2200$ meters. The distance between the two tunnels is $1000 \times 6 - 1200 - 2200 - 800 = 1800$ meters.

11. There are 30 cubes with side length 1 cm as shown in the figure below. Find the surface area of this solid figure, in cm^2.

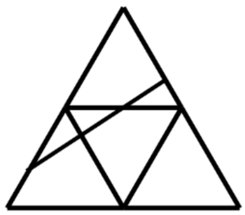


[Solution] The surface area is simple the area of six surfaces. We only need to multiply the front view, side view and top view by 2. The top view is a square with an area of 16. The area of both the side view and the front view is 10, so the total area is 72.

12. On a long stick, there are three types of scale lines. The first type of scale line divides the stick into 10 equal parts, the second type divides it into 12 equal parts, and the third type divides it into 15 equal parts. If the stick is sawed along each scale line, how many segments will the stick be divided into in total?

[Solution] The LCM of 10, 12, 15 is 60, so we can treat this long stick as having 60 units. The first, second and third type of scale divides this long stick into segments of 6, 5 and 4 units respectively. There are a total of $9+11+14=34$ scale lines on this long stick but some of them will overlap with each other. The LCM of 5, 6 is 30, so the scale lines will overlap once at this location. The LCM of 4, 5 is 20, so the scale lines will overlap at 20, 40. The LCM of 4, 6 is 12, so the scale lines will overlap at 12, 24, 36, 48. Therefore, there are $34-1-2-4=27$ scale lines on this long stick and it will be divided into 28 segments.

13. How many triangles are there in the figure below?



[Solution] By removing the additional line in the figure, there are 5 triangles. As the additional line is added into the figure, 5 more triangles are formed. $5+5=10$.

14. The following figure is a vertical algorithm, where the same letters represent the same numbers, and different letters represent different numbers. It is known that $\overline{BAD}$ is not a multiple of 3, and $\overline{GOOD}$ is not a multiple of 8. What is the four-digit number $\overline{ABGD}$?

$$\begin{array}{rcccc} & & B & A & D \\ + & & B & A & D \\ \hline & G & O & O & D \end{array}$$

[Solution] First, it can be determined that the value of D must be 0, and the value of G must be 1.

Therefore, $\overline{GOO} = \overline{BA} + \overline{BA}$ which mean $\overline{GOO}$ is an even number, which can only be 122, 144, 166, or 188. Since $\overline{BAD}$ is not a multiple of 3 and $\overline{GOOD}$ is not a multiple of 8, $\overline{GOO}$ is not a multiple of 3 and 4, we can eliminate 144 and 188. By testing 122 and 166, we can determine that only 166 is valid. At this point, $\overline{BAD}$ is 830, so the four-digit number $\overline{ABGD}$ is 3810 .

15. Find the sum of all proper fractions with prime denominators and the denominators are less than 30.

[Solution] The prime numbers that are less than 30 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29. Take denominator 17 as example, the sum of all proper fractions with 17 as denominator is:

$$(\frac{1}{17}+\frac{16}{17})+(\frac{2}{17}+\frac{15}{17})+(\frac{3}{17}+\frac{14}{17})+\cdots\cdots+(\frac{8}{17}+\frac{9}{17})=8=\frac{17-1}{2}$$

Similarly, we can find the sum of other proper fractions with different prime denominator, the sum is:

$$\frac{1}{2}+\frac{3-1}{2}+\frac{5-1}{2}+\frac{7-1}{2}+\frac{11-1}{2}+\frac{13-1}{2}+\frac{17-1}{2}+\frac{19-1}{2}+\frac{23-1}{2}+\frac{29-1}{2}$$
$$=\frac{1}{2}+1+2+3+5+6+8+9+11+14=59\frac{1}{2}$$

16. Among the natural numbers from 1 to 1999, how many of them, when added to 5678, result in at least one carry over?

[Solution] Among the natural numbers from 1 to 1999, how many of them, when added to 5678, do not result in any carry over? For such numbers, there are 2 possible options for the units digit (0 and 1), 3 possible options for the tens digit (0, 1, and 2), 4 possible options for the hundreds digit (0, 1, 2, and 3), and 2 possible options for the thousands digit (0 and 1). By the multiplication principle, there are a total of $2\times3\times4\times2=48$ numbers. But note that the calculation above includes the number 0 (0000). Therefore, among the natural numbers from 1 to 1999, there are $48-1=47$ numbers that, when added to 5678, do not result in any carry. Hence, among the natural numbers from 1 to 1999, there are $1999-47=1952$ numbers that, when added to 5678, result in at least one carry.

17. For a task, it takes 10 hours for student A following 10 hours for student B to complete it. If student A spends 8 hours following with student B spending 13 hours, they can still complete the task. Now, student A has already spent 2 hours, how many more hours do student A and B need to work together to complete the task? (Write your answer in hours as proper fraction).

[Solution] From the question, the combined efficiency of A and B is $\frac{1}{10}$. Student A spends 8 hours following with student B spending 13 hours is equivalent to A and B work together for 8 hours following with B works alone for 5 hours, so the efficiency of B working alone is $(1-8\times10)\div5=\frac{1}{25}$. It follows that the efficiency of A working alone is $\frac{1}{10}-\frac{1}{25}=\frac{3}{50}$. So, if A works 2 hours, it takes A and B $\left(1-\frac{3}{50}\times2\right)\div\frac{1}{10}=8\frac{4}{5}$ hours to complete the task.

18. The ratio of the money between person A and person B originally was 6:5. Later, person A received an additional 180 dollars, and person B received an additional 30 dollars. Now, the ratio of their money becomes 18:11. What is the sum of the original money amounts for person A and person B?

[Solution] The original ratio of money between two individuals is 6:5. If person A receives 180 dollars and person B receives 150 dollars, the ratio of their money remains unchanged at 6:5. Now, if person A receives an additional 180 dollars and person B receives only 30 dollars, which is 120 dollars less than before, the new ratio of their money becomes 18:11. This is equivalent to after person A's money increases by 180 dollars and person B's money increases by 150 dollars, the ratio of their money is 18:15. Then, when person B's money decreases by 120 dollars, the ratio of their money becomes 18:11. Therefore, 120 dollars is equivalent to 4 units, and 1 unit is equal to 30 dollars. The sum of their money after this is $30\times(18+15)=990$ dollars. Hence, the total sum of their original money is $990-180-150=660$ dollars.

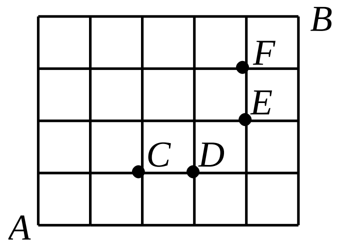
19. Legend has it that there is a country called the Land of Liars. In this country, men tell the truth on Thursday, Friday, Saturday, and Sunday, while they lie on Monday, Tuesday, and Wednesday. Women, on the other hand, tell the truth on Monday, Tuesday, Wednesday, and Sunday, but they lie on Thursday, Friday, and Saturday.
One day, a person visited the Land of Liars and met a man and a woman there. The man said, "Yesterday, I was lying." The woman said, "Yesterday was also a day when I told lies."
Now, the visitor is puzzled. What day of the week is it today? Based on their statements, can you determine the day of the week? [Write down Monday, Tuesday, Wednesday, etc. in your answer sheet]

[Solution] Assuming that the man is telling the truth today, it means that today is one of the days: Thursday, Friday, Saturday, or Sunday. Additionally, if the man was lying the day before, it means that according to his statement, today must be Thursday. Therefore, the woman is lying, yesterday is Wednesday which is precisely the day the woman tells the truth. Hence, today is Thursday.

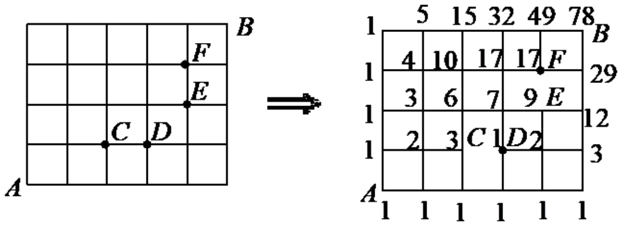
20. By swapping the tens and units digits of a two-digit number, we obtain a new two-digit number. If the difference between the original two-digit number and the new swapped number is 45, what is the largest possible value for the original two-digit number?

[Solution] $\overline{ab}-\overline{ba}=(10a+b)-(10b-a)=9(a-b)=45$ so $a-b=5$, to maximize the original number, $a=9$, $b=4$, so the maximum value is 94.

21. As shown in the figure, what is the number of shortest paths along the grid lines from A to B without passing through segments CD and EF?



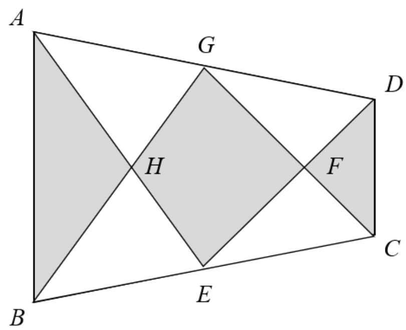
[Solution]



22. Class Kangaroo has 50 students. There are 28 students participating in the Chinese language competition, 22 students participating in the Mathematics competition, and 20 students participating in the English competition. If each student can participate in a maximum of two competitions, what is the maximum number of students in the class who have not participated in any competition?

[Solution] Due to the total number of participants being $28+22+20=70$, in order to maximize the number of students who have not participated in any competition, the number of participants should be minimized as much as possible. If all the participants took part in two competitions, the minimum number of participants is $70 \div 2 = 35$, which means the maximum possible number of students who have not participated in any competition is $50-35=15$.

23. As shown in the figure below, in the trapezoid *ABCD*, *AB* and *DC* are parallel, and the points *G* and *E* are the midpoints of *AD* and *BC* respectively. Given that the area of the quadrilateral *HEFG* is 14 cm^2, find the total shaded area in cm^2.



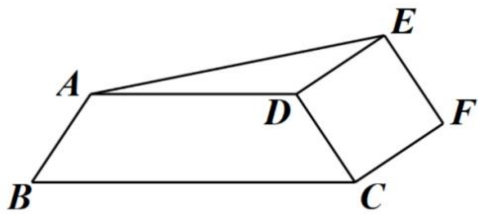
[Solution] We know that area of ΔBGC and ΔAED are both half of the area of the trapezoid *ABCD*. It follows that $AED+BGC+ABH+FDC=ABCD+HEFG$. From $BGC+AED=ABCD$, we have $ABH+FDC=HEFG$, so the total shaded area is $14\times 2=28$ cm^2.

24. The concentration of alcohol in bottle A is 70%, and the concentration of alcohol in bottle B is 60%. After

the two bottles of alcohol are mixed, the resulting concentration is 66%. If 5 liters are taken from each bottle and then mixed, the resulting concentration is 66.25%. How many liters of alcohol were originally in bottles A and B combined?

[Solution] First remove 5 liters from each bottle and mix together to get a 10 liters solution C of 65% concentration. Mix the remaining solution from bottle A and B to get a solution D with 66.25% concentration but unknown volume. If we now mix solution C and D, which is equivalent to mixing the original solution A and B, will obtain 66.25% concentration. From the crossing-method, this means that the volume ratio between solution C to D is 1 to 4. So, if solution C is 10 liters, then solution D is 40 liters. Again from the crossing-method, when mixing to get solution D, the ratio between solution A and B is 5 to 3, it shows that after removing 5 liters from each bottle, bottle B has $40\times\frac{3}{5+3}=15$ liters and bottle A has $40\times\frac{5}{5+3}=25$ liters. Originally, there are $15+25+10=50$ liters of alcohol combined.

25. In the figure, quadrilateral *CDEF* is a square, and quadrilateral *ABCD* is an isosceles trapezoid. Given that *AD* is 23 cm and *BC* is 35 cm, what is the area of triangle *ADE* in cm^2?

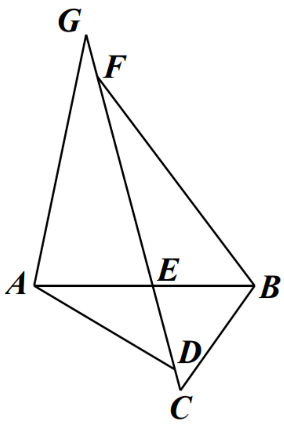


[Solution] By drawing the chordal diagram of the square *CDEF*, one can find that the height of the triangle *ADE* is $(35-23)\div 2=6$ cm. So, the area of triangle *ADE* is $23\times 6\div 2=69$ cm^2.

26. Five people, denoted as A, B, C, D, E, took a test with a maximum score of 100. Their scores are all distinct integers greater than 91. If the average score of A, B, and C is 95, and the average score of B, C, and D is 94, with A being the first place and E being the third place with a score of 96, what is the score of D?

[Solution] The total score of A, B and C is $95\times 3=285$. The total score of B, C and D is $94\times 3=282$. So, we see that A' score is $295-282=3$ greater than D's score. Since A is the first place and E is the third place with a score of 96, A's score can be 98 or 100. If A's score is 98, D's score is 95, the combined score of B and C is $285-98=187$. But since one from B or C must be the second place with score 97, the other one must be with a score of $187-97=90$ which violates the question. If A's score is 100, D is the second place with score 97, the combined score of B and C is $285-100=185$ which could be 92 and 94. The only possible score for D is 97.

27. In the figure below, points *D*, *E*, and *F* are on the line *CG*, *CD* = 2, *DE* = 8, *EF* = 20, *FG* = 4. The entire figure is divided into two parts by *AB*: the lower part has an area of 67, and the upper part has an area of 166. What is the area of triangle *ADG*?



[Solution] Connect AF and let the area of ΔAFG be $1u$. Since $GF:FE:ED=1:5:2$, the area of ΔAFE is $5u$ and the area of ΔAED is $2u$. The upper part of this figure has an area of 166, so the area of ΔFEB is $166-1u-5u=166-6u$. The lower part of this figure has an area of 67, so the area of ΔEBC is $67-2u$. From the ratio $FE:EC=2:1$, we have $2:1=(166-6u):(67-2u)$, so $1u=16$. So the total area of ΔGAD is $1u+5u+2u=128$.

28. A number X gives a remainder of 2 when divided by 3 and a remainder of 1 when divided by 4. What is the remainder when X is divided by 12?

[Solution] X gives a remainder of 2 when divided by 3 is equivalent to X gives a remainder of 3+2 when divided by 3. Similarly, X gives a remainder of 1 when divided by 4 is equivalent to X gives a remainder of 1+4 when divided by 4. This means that $X-5$ is divisible by both 3 and 4, so when X is divided by 12, one will obtain a remainder of 5.

29. For a cube, there are 8 vertices, 12 midpoints on the edges, 6 center points on the faces, and 1 center point of the cube, total of 27 points. Using any 3 points from these 27 points, how many triangles can be formed?

[Solution] When choosing three points out of the 27 points, either the three points are on the same line, or they can form a triangle. There is a total of $27\times26\times25\div(3\times2\times1)=2925$ ways to choose three points from the 27 points. The lines passing through the three points can be categorized into three types. There are lines formed by connecting two vertices, $8\times7\div2=28$ lines. There are lines formed by connecting two center points on the faces, 3 lines. And finally, there are lines formed by connecting two midpoints on the edges, $12\times3\div2=18$ lines. Therefore, $2925-28-3-18=2876$.

30. There are two docks, A and B, on a river, with dock A located 50 kilometers upstream from dock B. A passenger boat and a cargo ship depart from docks A and B at the same time, respectively, and both travels upstream. The passenger boat and the cargo ship have the same constant speed in still water. When the passenger boat departs, the luggage of a passenger falls into the water, and it floats along the current. 10 minutes after the passenger boat's departure, the parcel is 5 km away from the passenger boat. The captain of the passenger boat only realized this after it had travelled for 20 km and immediately turned around to look for the luggage. When the passenger boat reached the luggage, they also met with the cargo ship. Find

the speed of the river (in km/h).

[Solution] From the speed difference between the passenger boat in river and river, we can find the speed of the passenger boat in still water to be $5 \div \frac{1}{6} = 30$ km/h. Next, it takes the cargo ship $\frac{50}{30} = \frac{5}{3}$ hour to meet the luggage. Since we know if it takes the passenger boat time *t* to realize the luggage has been dropped, it will also take time *t* to retrieve the item, so under $\frac{5}{3} \div 2 = \frac{5}{6}$ hours, the passenger boat has travelled 20 km along the river. With the fact that the speed of the passenger boat in still water is 30 km/h, the speed of the river is found to be $30 - \left(20 \div \frac{5}{6}\right) = 6$ km/h.