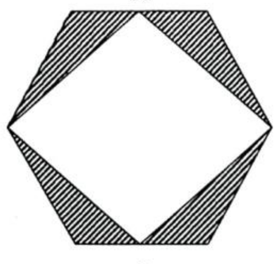
NMOS 2019 Special Round

Time Duration: 1.5 hour Name: __________ Marks: __________

1. Evaluate: $36\times\left(\frac{1}{1\times 6}+\frac{1}{6\times 11}+\frac{1}{11\times 16}+\frac{1}{16\times 21}+\frac{1}{21\times 26}+\frac{1}{26\times 31}+\frac{1}{31\times 36}\right)$

2. The operation $\langle A|B\rangle$ is defined as $\langle A|B\rangle = A\times(A+1)\times(A+2)\times\ldots\times(B-1)\times B$, where A is smaller than B, and A and B are whole numbers. For example, $\langle 3|8\rangle = 3\times 4\times 5\times 6\times 7\times 8$. What is the last digit of the sum $\langle 2006|2015\rangle + \langle 2016|2019\rangle$?

3. The figure below shows a regular hexagon with area 96 cm^2. A and B are the midpoints of the top side and the bottom side respectively. Find the total area (in cm^2) of the shaded regions.



4. A school planned to distribute Children's Day gifts among Class *A*, *B*, *C*, *D* and *E* in the ratio of 1: 2 : 3 : 4: 5 . However, on the actual day, the gifts were given out to Class *A*, *B*, *C*, *D* and *E* in the ratio of 6 : 7 : 8 : 9 :10 . As a result, one class received 5 less gifts than planned. The number of gifts that each class received is a whole number. Find the total number of Children's Day gifts that these five classes received.

5. Alice travels from Town P to Town Q and Bethany travels from Town Q to Town P along the same route in the opposite direction. Alice starts off her journey 15 minutes later than Bethany, and she arrives at her destination 30 minutes earlier than Bethany. Given that they travel at constant speeds, and Alice travels at twice the speed of Bethany. How many minutes have elapsed from Bethany's start time when they meet each other?

6. Given that $\dfrac{1}{\frac{2}{2017}+\frac{2}{2018}+\frac{2}{2019}} = P + q$, where P is a whole number and $0 < q < 1$, find the value of P.

7. Figure 1 shows the cross section of a rectangular can ABCD with base AB of length 60 cm. The tank is filled with water up to a depth $\frac{3}{4}$ of the height BC. When AB is tilted at 45°, as shown in Figure 2, the water level came up to C. Find the length (in cm) of the height BC.

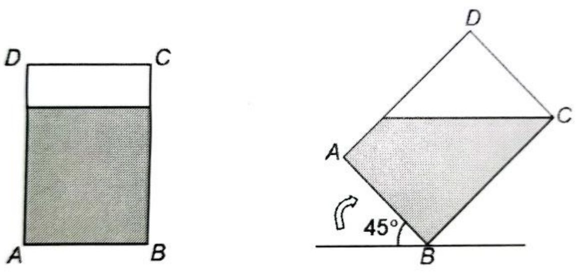
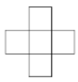
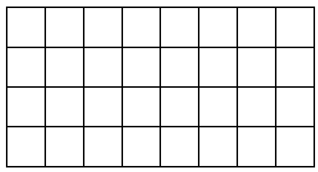


Figure 1 Figure 2

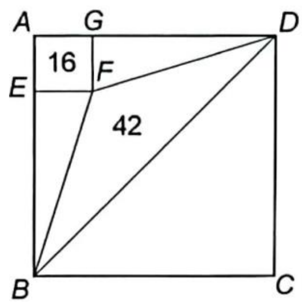
8. On the first day, Gopal travels from Bishan to Bedok at a constant speed of 49 km/h. On the second day, Gopal takes the same route. However, he needs to stop for a short while to refill his petrol. Consequently, he has spent an additional 8 minutes compared to the first day and travelled an additional 1 km for this detour to the petrol station. Given that his average speed for the journey on the second day is 45 km/h, how many minutes did Gopal take to travel from Bishan to Bedok on his first day?

9. Let A be a whole number. If the value of the fraction $\frac{A+23}{A-37}$ is also a whole number, how many different possible values of A are there?

10. One needs at least two cross shapes  to cover a 2×2 chessboard, where overlapping and extrusion are allowed. Find the smallest number of such cross shapes needed to cover a 4×8 chessboard.



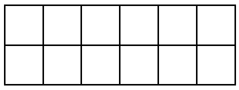
11. In the following figure, both ABCD and AEFG are squares. If the area of square AEFG is 16 cm^2 and the area of triangle BFD is 42 cm^2, find the area (in cm^2) of square ABCD.



12. Let A and B be two positive whole numbers such that $A+B$ and A^2+B^2 are both multiples of 7. If A is a 2-digit number and B is a 4-digit number, find the largest possible value of $B-A$.

13. Find the total number of even numbers from 1 to 2019 that is either a multiple of 8 or is not a multiple of 4.

14. Find the total number of ways to pave a 2×6 block with 6 tiles of the size 1×2, assuming tiles of the same size are indistinguishable.



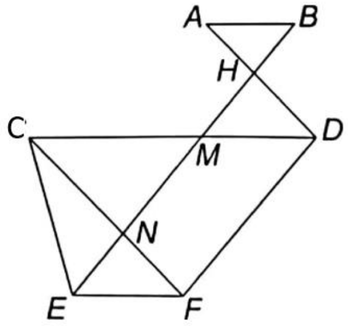
15. For a positive whole number n, the expression n! (read as n factorial) is defined as follows:

$$n! = (n)\times(n-1)\times(n-2)\times\ldots\times 2\times 1$$

For example, $3! = 3\times 2\times 1$ and $5! = 5\times 4\times 3\times 2\times 1$.

Find a 3-digit positive whole number, whose value is equal to the sum of the factorials of each of its digit.

16. In the figure below, *AB*, *CD* and *EF* are parallel to each other. *AD* is parallel to *CF*, $BH = 1cm$, $HM = 2cm$, $MN = 3cm$ and $EN = 2cm$. If the area of quadrilateral *CDFE* is *m* times the area of triangle *ABH*, whereby *m* is a whole number, find the value of *m*.



17. John played a game. First, he wrote down the number $\frac{4}{5}$ on the white board. In each turn, he tossed a coin: if he got a Head, he doubled the number on the board; if he got a Tail, he replaced the number on the board by its reciprocal. After 2019 turns, John noticed that the number on the white board was $\frac{2}{5}$. Find the largest possible number of Heads he tossed.

18. A fortune teller has a magic crystal ball. It shines upon a string of digits written on the paper and make the following changes: keep the first digit unchanged and replace the rest of the digits by their sum.

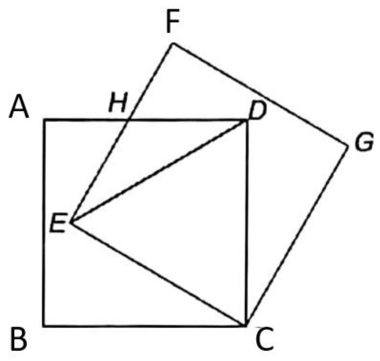
For example, if the crystal ball shines upon the number 0237, a new number 012 will appear on the paper and the old number is erased, where $12 = 2 + 3 + 7$.

The fortune teller will ask every visitor to write down their birth date in the format of *ddmmyyyy*. For example, those who were born on the 19th March 2004 will write down 19032004.

The fortune teller will use his crystal ball to shine upon this 8-digit number, obtain a new number, use the crystal ball to shine upon this new number, obtain the second new number, and repeat to obtain the third (and final) number. This final number is called the *magic code* of that visitor.

Find the total number of possible distinct magic codes the fortune teller may give to the visitors.

19. In the figure below, the square *ABCD* overlaps with the square *CEFG*. Both squares have the same area 100 cm^2. Given that the area of the hexagon ABCGFH is 125 cm^2, find the length (in cm) of *DE*.



20. If *a*,*b*,*c*,*d*,*e*,*f*,*g*,*h* and *j* represent different whole numbers from 1 to 9 (such that $a+\frac{b}{c}+\frac{d}{e}\times f-\left(g+\frac{h}{j}\right)=N$ where *N* is a whole number. Find the largest possible value of *N*.