NMOS 2022 Special Round

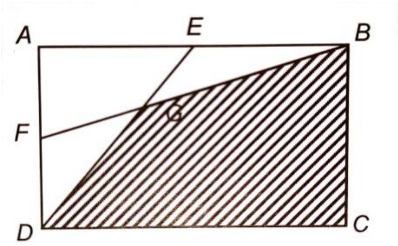
Time Duration: 1.5 hour Name: __________ Marks: __________

1. Find the value of $2000\times\left(1-\frac{1}{2}\right)\times\left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{4}\right)\times\ldots\times\left(1-\frac{1}{100}\right)$

2. For any positive integer *m*, let $d(m)$ denote the number of positive factors of *m*. For example, $d(1)=1$, $d(4)=3$ and $d(6)=4$.
   Let *X* and *Y* be the smallest three-digit and four-digit positive integers respectively such that $d(X)=d(Y)=d(2022)$. Find $X+Y$.

3. The average age of Andy, Benny and Calvin is 42 years old. If Andy's age is increased by 7 years old, and Benny's age is doubled, and Calvin's age is halved, all three will be of the same age. What is Calvin's current age?

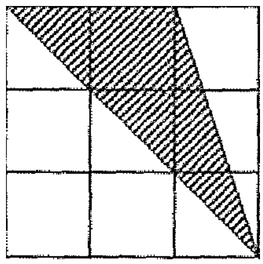
4. In the rectangle *ABCD* below, the points *E* and *F* are the midpoints of *AB* and *AD* respectively. *AE* = *EB* = 5 cm and *AF* = *FD* = 3 cm. *BF* and *DE* intersect at the point *G*. Find the area of the shaded region in cm^2.



5. A worker will work for 8 consecutive days before he takes 2 consecutive rest days. This week, his rest days fall on Saturday and Sunday. How many weeks later can he rest on Sunday again?

6. Let A be a two-digit number. It is known that the sum of A and 17 is a multiple of 5 , and the difference of A and 17 is a multiple of 6 Find the largest possible value of A.

7. In the following 3 by 3 square grid, the area of each 1 by 1 square is 1 unit^{2x} . How many triangles could be formed by 3 points from the 16 vertex points such that its area would be the same as that of the shaded region?



8. We define $a * b = \dfrac{7a - b}{7a + b}$ for any integers *a*, *b* where $7a + b \neq 0$.

Find the sum of all positive integers *n* such that $n * (n^2 * n^3) = 1$?

9. There are 12 machines in Warehouse A and 7 in Warehouse B. These machines are to be relocated in Factory I and II: 9 to be sent to Factory I, and 10 to Factory II. The unit cost of transporting one machine is listed as follows.

Cost of transporting one machine	To Factory I	1 To Factory II
From Warehouse A	50	100
From Warehouse B	35	70

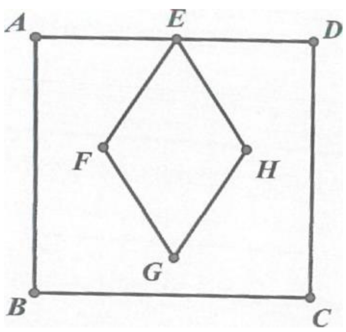
Find the minimal total cost needed, in S$, to transport all the machines.

10. Given a two-digit positive integer $\overline{ab}$, we define $P\left(\overline{ab}\right)$ as the product of *a* and *b*. For example, $P(23)=6$ and $P(54)=20$. How many two-digit positive integers $\overline{ab}$ are there such that $\overline{ab} > \left(P\left(\overline{ab}\right)\right)^2$?

11. Cars X and Y travelled from Station A to Station B starting at the same time. During the first 10 minutes, car X was faster than car Y by 2.5 km per hour. After travelling for 10 minutes, car X decreased its speed. After another 5 minutes, car Y decreased its speed to the extent that it was slower than car X by 0.5 km per hour. They travelled for another 25 minutes without changing speed and reached Station B at the same time. Find the decrease in speed of car X in km per hour.

12. A fire dragon is guarding the treasury and the only way to enter the treasury safely is as follows. First, one must shout a positive integer M to the dragon. The dragon will shout back $(M-13)^2+25-M$. Only if the two numbers differ by exactly 2, would one be allowed to enter the treasury. Otherwise, the dragon will attack! Find the smallest positive integer M which grants safe entry to the treasury.

13. The following is the road map of a national park, where the roads consist of a square ABCD with side length 7 km, and a rhombus EFGH with side length 4 km. E is the midpoint of AD. The management office is to choose a location around the roads and set up a fire station. A fire truck will be ready at the fire station with the speed p km per hour, where p is a positive integer. It is required that the fire truck must be able to reach any point along the roads within 9 minutes. By choosing a suitable location for the fire station, find the smallest possible value of p.



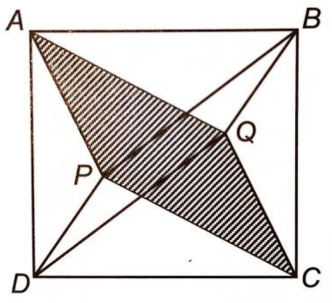
14. The following shows an ancient manuscript found where most of the digits could not be recognized and are represented by $*$. It is also known that the distinct letters represent distinct digits. Find $A+B+C+D$.

$$
\begin{array}{r}
**7 \\
*7\overline{)ABCCD} \\
\underline{***} \\
7* \\
\underline{**} \\
*** \\
\underline{***} \\
0
\end{array}
$$

15. 2020 students stand in a circle, each carrying a number 1, 2, ..., 2020 respectively in the clockwise direction. The counting starts from the first student (No. 1) and proceeds as A, B, C, D, E, A, B, C, D, E, ... around the circle clockwise. Students who are counted with E will stay and every other letter will leave the circle. For example, Student No.1, 2, 3 and 4 will leave the circle and Student No. 5 stays. The remaining students form a smaller circle. If this counting continues along the subsequent circles, what is the number of the last student who stays in the smallest circle?

16. Two outposts A and B built along the river are 270 km apart, where water flow from A to B. In the morning, a fast ship travels from A to B and at the same time, a low ship travels from B to A. The two ships are scheduled to meet at 90 km from B. Their speed in still water differs by exactly the same value as the water flow speed. However, after 2 hours, the fast ship malfunctions and travels downstream carried forward by only the water flow. As a result, the two ships meet at 135 km from B instead. Find the water flow speed in km per hour.

17. In the following square *ABCD*, *P* and *Q* are interior points such that $\frac{[ABP]}{[DPC]}=\frac{3}{2}$, $\frac{[ADP]}{[BCP]}=\frac{3}{7}$, $\frac{[ABQ]}{[CDQ]}=\frac{3}{5}$ and $\frac{[ADQ]}{[BCQ]}=4$. If $\frac{[APCQ]}{[ABCD]}=\frac{m}{n}$ where $\frac{m}{n}$ is a fraction in lowest terms, find the value of $m+n$. ($[ABCD]$ denotes the area of rectangle *ABCD*)



18. In the following 3×3 square grid, all rows, columns and the two diagonals will sum to the same value. Find the value of number *A*.

A	35	89
1		

19. Given that in a set of *n* three-digit numbers, there must be at least 3 numbers such that their sum of digits are the same. Find the least possible value of *n*.

20. In the following right-angled trapezium, point *E* lies on *AB* such that ΔCDE is an isosceles right-angled triangle. Given that $BE = 20\,cm$ and $CE = 52\ cm$, find the area of ΔADE in cm^2.