

Solution

$$1. 3145 \times \frac{2 \times 3 \times 4 \times (1+2 \times 2 \times 2+3 \times 3 \times 3)}{4 \times 5 \times 6 \times (1+2 \times 2 \times 2+3 \times 3 \times 3)} = 3145 \times \frac{1}{5} = 629$$

2. The age difference between Alex and his father remains the same at 28 years old. After several years, his father's age is three times that of Alex and 28 years older than Alex. Assuming that Alex's age is one part, then his father's is three parts. So, 28 years are two parts. One part is 14 years. And his father is three parts this year, which is 42 years old.

3. If several odd numbers are added from 1, the sum is the square of the number of terms.

$$1+3+5+7+9+11+13+\dots+31=16^2=256, \text{ so original formula } =256-1-3=252$$

4. Let the quantity of the ordinary edition and the signed edition be X and Y respectively, and the total price is 213 yuan. $19x+40y=213$. Because $40y$ is a whole-tens number, 213's units digit is obtained from $19x$'s units digit. So, x is 7. And the number of signed versions is 2.

5. 49 is an odd number, so one of A and B is an odd number and the other is an even number. 60 is even, so B and C are both odd numbers or even numbers. Of all prime numbers, 2 is the only even number, so we can find $a=2$, and $b=47$, $c=13$.

6. Alex's individual work efficiency is $\frac{1}{12}$. After 8 days of work, two-thirds of the workload was completed. So, the remaining one third of the work was completed by Ben in 10 days. From this, we can know the efficiency of Ben's independent work is $\frac{1}{30}$. And then we can get that Ben would complete independently in 30 days.

$$7. v_{boat} + v_{river} = 352 \div 11 = 32 \text{ km/h}; \quad v_{boat} - v_{river} = 352 \div 16 = 22 \text{ km/h}, \\ v_{boat} = (32 + 22) \div 2 = 27 \text{ km/h}$$

8. According to E and G are the midpoint of the legs, it can be seen that both the $\triangle AGB$ and the $\triangle DEC$ are half of the whole trapezoid. $S_{\triangle AGB} + S_{\triangle DEC} + S_{\triangle AHD} + S_{\triangle FBC} = S_{\text{trapezoid}ABCD} + S_{HEFG}$. So, $S_{\triangle AHD} + S_{\triangle FBC} = S_{HEFG}$. And $S_{HEFG} = 18 \div 2 = 9$.

9. $1011 \times A = A \times B \times \overline{BBA}$. A isn't 0. So, $1011 = \overline{BBA} \times B$. According to the result's units digit, $1, A \times B$ is 1×1 or 3×7 . Because the result is 1011, we can know $A=7, B=3$. $\overline{AB} = 73$.

10. The total number of words is invariable. The ratio of the number of words recited in the first 3 days to the number of words recited in the next 4 days is $3:4$. 7 parts in total. The ratio of the

number of words recited in the next three days to the number of words recited in the previous four days is $5:6$. 11 parts in total. If the total amount is unified into 77 parts, the fourth day's accounts for $\frac{9}{77}$. Since the total amount is between 100 and 200 and is a multiple of 77, the total amount is

154. So, number of words recited in the 4th day is $154 \times \frac{9}{77} = 18$.

11. Through the analysis of the topic, it is found that the surface area is the area of six surfaces. You only need to multiply the main view, left view and top view by 2. The top view is a square with an area of 16. The area of both the left view and the main view is 10, so the total area is 72.

12. If the sum of two numbers is certain, the smaller the difference, the larger the product. Divide 110 into 43 and 67, and the maximum product is 2881.

13. The method is backward induction. The final result of this question is 50. In order to take as few steps as possible, we choose to multiply the maximum number by 2. And then the number in the previous step is 25. Then according to the same method. Since 25 is an odd number, it can only be obtained by adding or subtracting 3 from the previous number. So, the previous number may be 22 or 28. Because there will be side effects after using the super potion, the previous number is 28. And then it can be pushed back step by step according to the same method.
 $50 \rightarrow 25 \rightarrow 28 \rightarrow 14 \rightarrow 11 \rightarrow 8 \rightarrow 5 \rightarrow 2$. Therefore, a total of 7 bottles are required.

14. If we average the ages of A and B teams, the average age of A will change from 37 to 29, a decrease of 8. The average age of B changed from 23 to 29, an increase of 6. According to the nature of the average number, the increased part is the same as the decreased part. So, the number ratio of the two teams is $6:8$. Similarly, after averaging B and C, the average age of B changed from 23 to 33, an increase of 10. The average age of C changed from 41 to 33, decreased by 8. So, the number ratio of B and C is $8:10$. The total ratio is $6:8:10$, that is $3:4:5$. Assuming 3 persons in group A, 4 persons in group B and 5 persons in group C, the average age is
 $(3 \times 37 + 4 \times 23 + 5 \times 41) \div (3+4+5) = 34$

15. Some triangles in this question are large and some are small, so each small piece will be numbered separately. Then count from less to more according to the number of pieces: first count the triangles consisting of only one piece, and there are 5 triangles. Then count the triangles consisting of only 2 pieces, there are 4. Count the triangles consisting of only 3 pieces, there are 2. And all of five small triangles consist the largest triangle. Finally, add all the results to get 12.

16. The total number of gold medals and bronze medals is 75% more than that of silver medals, that is, the total number of gold medals and bronze medals is $\frac{7}{4}$ of that of silver medals. And it is $\frac{7}{11}$ of the total number. The total number of gold medals and bronze medals is 56. Because the number of gold medals is 20 more than that of bronze medals, the number of bronze medals is
 $(56 - 20) \div 2 = 18$

17. According to the same speed sum of the two people before and after the encounter, it can be concluded that the time ratio is equal to the distance ratio. The ratio between the whole journey and the journey after the encounter is $12:8$, which is $3:2$. If the distance difference between the

two is 100km, then 1 part is 100km. And the whole journey is 3 parts, which is 300km.

18. Since the number of apples is 5 more than the number of peaches. It can be seen that when each person has 11 apples, there is still one apple left. When each person has 10, there are 8 left. So, each person allocates one more, and the total number is 7 less. We can find that the number of people is 7. So, the number of apples is $7 \times 10 + 8 = 78$ and the number of peaches is $7 \times 11 - 3 = 74$. The total number is 151.

19. According to the characteristics of “palindromic number”, it is found that the sum of 2022 can only be obtained by adding four-digit number and three-digit number. According to the hundreds digit is 0, it can be judged that there must be a carry-over between thousands digit and hundreds digit. Then the thousands digit of the four-digit number is 1, then the units digit is 1 too. And the units digit of the result is 2, so the units digit and hundreds digit of the three-digit number are also 1. Next, we will find the numbers of hundreds digit and tens digit. From the carry analysis, it is easy to get that the four-digit number’s hundreds digit and tens digit are 8. And the three-digit number’s tens digit is 4. So the difference is $1881 - 141 = 1740$.

20. According to the “deformation between equal area” (the area of the triangle with the same bottom and equal height between parallel lines is equal), it can be obtained that the area of $\triangle DEG$ is equal to the area of $\triangle BEG$. Similarly, the area of $\triangle EGK$ is equal to that of $\triangle EGF$, so the area of the shadow is equal to the area of BEFG. And the calculated result is 64 cm^2 .

21. Finally, there are two numbers left. It shows that 629 is the number left after erasing. The number we are looking for is the latter number written. According to the meaning of the question, the number written should be the remainder of the number erased in the previous step divided by 11. From the extreme point of view, suppose that the number erased is the number other than 629. From the additivity of the remainder, the remainder of the sum of 1 to 2022 divided by 11 is 1. The remainder of 629 divided by 11 is 2. Similarly, by using the nature of the remainder, it can be seen that the remainder of the sum of the remaining numbers divided by 11 should be 10.

22. According to the meaning of the question, the multi-digit number is a multiple of the upper left corner. In combination with the multiple characteristics of 2, 3, 4 and 5, give priority to 5. So that the answer in row 5 is 12345. Then look at row 4. Fill in 1 or 2 for the four-digit’s units digit in the dotted line box. Because it is a multiple of 2, the units digit is 2. Then analyze the remaining spaces according to the non-repetition of each row and column and the filled spaces. As shown in the figure below, the fourth row 41523.

3	5	2	1	4
² 5	4	1	3	2
³ 2	3	4	5	1
⁴ 4	1	5	2	3
⁵ 1	2	3	4	5

23. According to the condition of regular pentagon, an internal angle of regular pentagon is 108° . Then, according to the symmetry, CG and DF are both angular bisectors. So $\angle HCD$ and $\angle HDC$ are both 54° , then x can be 72° .

24. Alex, Cindy and Elsa must be bigger than Ben. So Ben must be 1 or 2. We can assume that Ben is 1. Alex is not adjacent to Ben, so it is 3 or 4. If it is 4, Elsa can only be 5, which contradicts the condition. So Alex is 3, Elsa and Alex are not adjacent. Elsa is 5. David and Elsa are not adjacent, so David is 2. And Cindy is 4. The verification is valid, so Alex, Ben, Cindy, David and Elsa are 3, 1, 4, 2 and 5 respectively.

25. Move the yellow piece of paper to the left, and the sum of the yellow and green areas remains the same. After translation, the whole square box bottom is divided into four areas. The area of the red area remains unchanged. The areas of yellow and green areas are equal. Since the sum of the yellow and green areas is $18+12=30$, the yellow and green areas after translation are 15 respectively, and the blank area can be obtained according to the equal product of the area of the diagonal rectangles: $15 \times 15 \div 25 = 9$, Then add the four areas to get the square area. $25 + 9 + 15 + 15 = 64$.

26. The height of the two triangles in $\triangle ADC$ is equal. As long as the ratio of AH and HC is known, the ratio of the area of $\triangle DAH$ to $\triangle DCA$ can be known. The $\triangle AHE$ and $\triangle CHD$ satisfy the similarity relationship, and the similarity ratio can be obtained by using the known side length as $(24 + 60) : 60 = 7 : 5$. It can be seen that the ratio of AH and HC is also 7:5, so the shadow triangle accounts for $\frac{7}{12}$ of $\triangle DCA$. So, $60 \times 60 \div 2 \times \frac{7}{12} = 1050 \text{cm}^2$.

27. Every 2 columns (7 numbers) is a cycle. $2022 \div 7 = 288 \dots 6$. So, 2022 is the sixth number after the 288th cycle. The sixth number in a cycle should be in the second row of the second column.

28. From 629 to 699, there are 8 numbers containing 2. From 700 to 1999, there are 19 numbers containing 2 in units digit or tens digit every 100 numbers. There are 100 numbers whose hundreds digits are 2 from 1200 to 1299, and the calculation is repeated. The 19 numbers in this range should be removed. From 2000 to 2022, there are 23 numbers contain 2 in thousands digit. So, $8 + 19 \times (13 - 1) + 100 + 23 = 359$.

29. Take the middle point I of the bottom edge. Connect the point G and the point I, and intersect EF at point J to divide the whole figure into four identical parallelograms. So, $11 - S_{JOHI} = 7 + S_{\triangle GJO}$. In $\triangle GIH$, O is the midpoint of GH, so $\triangle GJO : \triangle GIH = 1 : 4$. And $\triangle GJO = 1$. So the area of the small parallelogram is $7 + 1 = 8$, the area of the large parallelogram is 32, and the area of the shaded part is $32 - 7 - 11 = 14$.

30. The sum of the four cycles is certain. If the central cycle is assumed to be A, then the sum of the three products is $A \times (50 - A)$ after extracting the common factor A. According to the sum is certain, the difference is small and the product is large. At $A = 25$, the product is the largest, which is 625.