NMOS Special Round Mock Test

Solutions

1. Find the value of $100\times(1-\frac{1}{2})\times(1+\frac{1}{3})\times(1-\frac{1}{4})\times(1+\frac{1}{5})\times...\times(1-\frac{1}{100})\times(1+\frac{1}{101})\times(1-\frac{1}{102})$

【Solution】 50

If all the "+" in the brackets become "-", calculation would be easy.

$$100\times(1-\frac{1}{2})\times(1+\frac{1}{3})\times(1-\frac{1}{4})\times(1+\frac{1}{5})\times...\times(1-\frac{1}{100})\times(1+\frac{1}{101})\times(1-\frac{1}{102})$$

$$=100\times(1-\frac{1}{2})\times(1-\frac{1}{3})\times\frac{4}{2}\times(1-\frac{1}{4})\times(1-\frac{1}{5})\times\frac{6}{4}\times...\times(1-\frac{1}{100})\times(1-\frac{1}{101})\times\frac{102}{100}\times(1-\frac{1}{102})$$

$$=100\times(1-\frac{1}{2})\times(1-\frac{1}{3})\times(1-\frac{1}{4})\times(1-\frac{1}{5})\times...\times(1-\frac{1}{100})\times(1-\frac{1}{101})\times(1-\frac{1}{102})\times\frac{4}{2}\times\frac{6}{4}\times\frac{8}{6}...\times\frac{102}{100}$$

$$=100\times(\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times...\times\frac{99}{100}\times\frac{100}{101}\times\frac{101}{102})\times(\frac{4}{2}\times\frac{6}{4}\times\frac{8}{6}...\times\frac{102}{100})$$

$$=100\times(\frac{1}{102})\times(\frac{102}{2})=50$$

2. The total of the ages of four people, Amy, Bill, Calvin and Danny, is exactly 100 this year. If Amy's age is increased by 4 years old, Bill's age is doubled, Calvin's age is halved, and Danny's age becomes only one-third of his current age, all four people will be of the same age. What is Danny's current age?

【Solution】 48

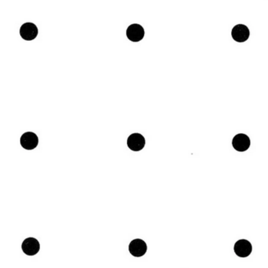
Suppose Bill's age is 1 unit, then Calvin's age is 4 units, Danny's age is 6 units and Amy's age is 2 units minus 4. The sum of these four is 13 units minus 4, which is equal to 100, indicating 1 unit is $(100+4)\div13=8$. Danny's current age is 6 units, which is $6\times8=48$.

3. Originally 25% of the students in the room were boys. Later, two boys enter this room and currently 28% of the students in the room are boys. How many girls are there in this room?

【Solution】36

The number of girls in this room remains unchanged, suppose it is “x”. Their percentage was 75% at first and 72% in the end. $x \div 75\% + 2 = x \div 72\%$. This means $x \times \frac{4}{3} + 2 = x \times \frac{25}{18}$. The number of girls is equal to $2 \div (\frac{25}{18} - \frac{4}{3}) = 2 \div \frac{1}{18} = 36$.

4. The following nine dots form a 2×2 grid. How many triangles can be formed using these dots?



【Solution】76

Any 3 points can form a triangle as long as they do not lie on the same straight line.

There are only 8 cases where 3 dots in this diagram rest on the same line:

(1) The four edges of the big square.
(2) Two diagonals, one vertical line and one horizontal line passing the middle dot.

Hence, the answer is $C_9^3 - 8 = 76$

5. A project can be done by team A alone in 20 days. It can also be done by team B alone in 15 days. If team A works on this project for several days and then leaves it to team B, from start to finish the completion of this project takes a total of 16 days. During these 16 days, how many days are done by team B?

【Solution】 12

View the project as "1". Team A and B has an efficiency of $\frac{1}{20}$ and $\frac{1}{15}$ respectively.

Method 1: Chicken and Rabbit in the Same Cage Problem

Suppose all 16 days are done by team A only, then $1-\frac{1}{20}\times 16=\frac{1}{5}$ of the project is left unfinished.

$\frac{1}{5}\div(\frac{1}{15}-\frac{1}{20})=12$ days are done by team B.

Method 2: Algebra

Suppose team B works for x days and team A thus works for $16-x$ days.

$\frac{1}{15}x+\frac{1}{20}(16-x)=1$ indicates $x=12$

6. Car A, B and C all start out at the same time from place X towards place Y. Meanwhile, a truck starts out from place Y heading towards place X. All four vehicles have constant speed. After 5 hours, 6 hours and 8 hours, the truck meets car A, B and C respectively. If car A's speed is 60 km/h and car B's speed is 48 km/h, find out car C's speed in km/h.

【Solution】 $33km/h$

When car A and the truck meet, A travelled $9-0=9$ while the truck travelled for 5 hours.

Their total travelling distance is XY.

When car B and the truck meet, B travelled $6\times 48=288km$ while the truck travelled for 6 hours.

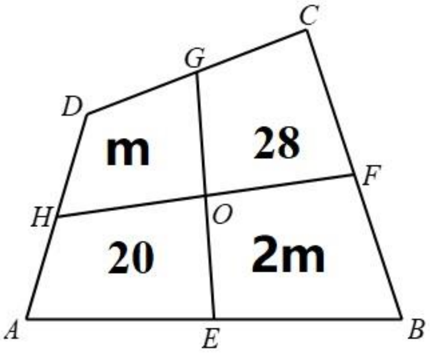
Their total travelling distance is XY.

This means in the extra one hour, the truck travelled $300-288=12km$.

The speed of the truck is $12km/h$. The distance XY is $5\times(60+12)=360km$.

Car C and the truck take 8 hours to meet. The sum of their speed is $360\div 8=45km/h$ and therefore the speed of car C is $45-12=33km/h$.

7. As shown below, point *E, F, G* and *H* are the midpoints on the four sides of the quadrilateral *ABCD*. EG and FH divide the quadrilateral into 4 regions, whose areas are known to be *m*, 28, 20 and *2m*. Find the value of *m*.



【Solution】16

Using Half-Model, we have m+2m=28+20 and hence m =16.

(Link OA, OB, OC and OD, due to Equal-Height Model, we obtain 4 pairs of equal triangles.)

8. The teacher bought some candies and planned to give them all out to A, B, C in the ratio of 3:4:5. However, on the actual day, the teacher gave all the candies out to A, B, C in the ratio of 5:6:7 instead. As a result, one child has received 2 more candies than planned. How many candies did the teacher buy in total?

【Solution】72

The unchanged quantity is the total number of candies. $3+4+5=12$ while $5+6+7=18$. 12 and 18 has lcm of 36. Hence 3:4:5 can be written as 9:12:15 and 5:6:7 can be written as 10:12:14. By comparison, A is the child that has received 2 more candies. 1u = 2 while total candies are 36u. 36u=72.

	A	B	C	total
Planned ratio	9	12	15	36
Actual ratio	10	12	14	36

9. A 7-digit number $2023\overline{ab}0$ is divisible by 99, where a and b represent different digit. Find the product of a and b.

【Solution】30

99 is the product of 9 and 11. $2023\overline{ab}0$ must be divisible by both 9 and 11.

(1) Divisible by 9: The sum of digits $2+2+3+a+b=7+a+b$ must be multiple of 9.
$a+b=$ 2 or 11.
(2) Divisible by 11: $(2+2+a+0)-(0+3+b)=a-b+1$ must be multiple of 11.
$b-a=1$ (10 is impossible since the largest difference is $9-0=9$)

The difference and the sum must be odd at the same time. y.

With both difference and sum known, we have $a=5$, $b=6$, the product $a\times b=30$.

10. Given that x and y are whole numbers such that $\frac{1}{x}-\frac{1}{2y}=\frac{1}{10}$, find the largest value of $x+y$.

【Solution】54

$\frac{1}{x}>\frac{1}{10}$, and thus $1\le x\le 9$. x has limited choices.

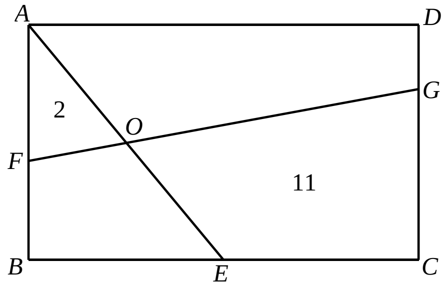
Method 1: Try all the possible values for x and compare. When $x=9$, $y=45$, $x+y$ is maximized.

Method 2: Find the relation between x and y.

x is larger $\rightarrow\frac{1}{x}$ is smaller $\rightarrow\frac{1}{x}-\frac{1}{10}$ is smaller $\rightarrow\frac{1}{y}$ is smaller $\rightarrow$ y is larger

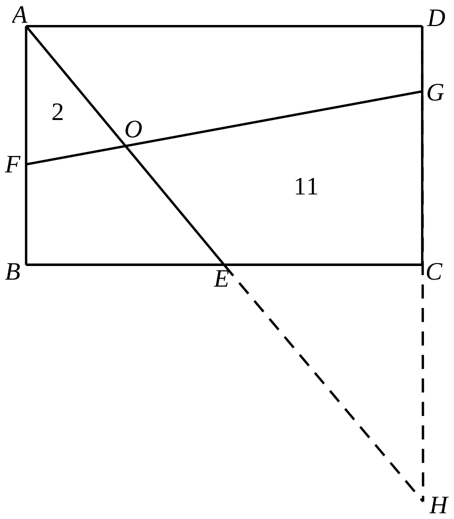
Hence when x reaches its maximum at 9, y is also maximized. The answer is 9+45=54.

11. As shown below, the line segments *AE*, *FG* divide the rectangle *ABCD* into 4 regions. Two of the areas are known to be 2 and 11. *E* is the midpoint of *BC*. The ratio of *AO: OE* is 2:3. Find the area of the rectangle *ABCD*.



【Solution】84

Extend AE and DC such that they meet at point H. E is midpoint, indicating $\triangle ABE$ and $\triangle HCE$ are the same. If AO is 2u, OE is 3u, then EH=AE=5u. In the Hourglass Model $\triangle AOF$ and $\triangle HOG$, AO:OH=2:(3+5)=2:8=1:4, and hence the area ratio is $1^2:4^2=1:16$. $\triangle HOG$ has an area of 32. $\triangle ECH$ has an area of $32-11=21$. $\triangle ABE=21$. $\square ABCD=21\times 4=84$.



12. There are in total 252 bars of chocolate. A natural number *n* needs to be chosen such that if Melvin eats *n* bars every day, the chocolate will be finished in a whole number of days. If *n* must be a prime number, there are *a* different choices of *n*. If *n* must be odd, there are *b* different choices of *n*. Find the value of $a+b$.

【Solution】9

$252=2^2\times 3^2\times 7$, which has 3 prime factors and $(2+1)\times(1+1)=6$ odd factors.

$a=3, b=6, a+b=9$

13. In the following 3×3 square grid, all rows, columns and the two diagonals will sum to the same value. Find the value of number M.

	M	
		2
11	40	

【Solution】M=20

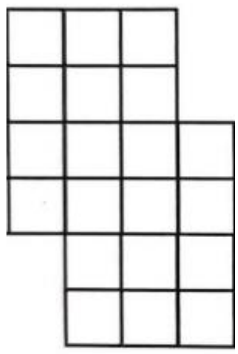
The upper right corner equals $BF = FG = GC = 2u$. Hence $M + 40 = 49 + 11$. $M = 20$.

14. Within one hour, there are two instances between 6 o'clock and 7 o'clock when the minute hand and the hour hand of the clock make an angle of 110 degrees. What is the time difference, in minutes, between these two timings?

【Solution】40 minutes.

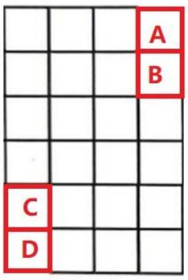
The minute hand moves from 110 degrees behind the hour hand to 110 degrees past the hour hand. $110 \times 2 \div 5.5 = 40\,\text{min}$

15. How many rectangles (squares are included) are there in the figure below?



【Solution】126

In the complete 6×4 grid, there are $C_7^2\times C_5^2=21\times10=210$ rectangles.



The rectangles containing one or more of A, B, C and D need to be deducted.

First consider only A and B: Rectangles containing

(1) A only: $1\times1\times4\times1=4$
(2) B only: $1\times5\times4\times1=20$
(3) Both A and B: $1\times5\times4\times1=20$

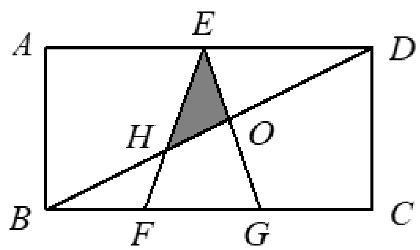
Hence rectangles containing A or B: $4+20+20=44$

Symmetrically, rectangles containing C or D: 44

Rectangles containing A/B as well as C/D: 4 (ABC, ABCD, BC and BCD)

The answer is $210-44-44+4=126$.

16. Rectangle $ABCD$ has an area of 70. Point E is the midpoint of AD. Point F and G divides the side BC into three equal parts. Find the area of $\triangle EHO$.



【Solution】3

Suppose $AD=BC=6u$, then $AE=ED=3u$ and $BF=FG=GC=2u$.

Use the Hourglass Model twice ($\triangle EOD$ and $\triangle GOB$; $\triangle EHD$ and $\triangle FHB$) to find that $ED:BG=3:4$ and $ED:BF=3:2$. Hence $EO:EG=3:7$ and $EH:EF=3:5$.

Then use Bird-Head Model in $\triangle EHO:\triangle EFG=\dfrac{EH\times EO}{EF\times EG}=\dfrac{3\times3}{5\times7}=\dfrac{9}{35}$.

Besides, $\triangle EFG:\square ABCD=1:6$. Hence, $\triangle EHO=70\times\dfrac{9}{35}\times\dfrac{1}{6}=3$.

17. The password to a suitcase is a 3-digit number.
A says: "It is 954." B says: "It is 214." C says: "It is 358."
The owner of the suitcase comments: "Each of you have got exactly one digit right, and the digits you guys guessed correctly are different from each other." What is the real password?

【Solution】918

A and B both guessed that the last digit is "4" while they got different digits right. Hence neither of them got the last digit right. It must be C that got the last digit right. Last digit is "8". The other two digits that C guessed must be wrong. Second digit is not "5". Hence A guessed the last two digits wrongly and the first digit is indeed "9". B got the second digit right, which is "1". The answer is 918.

18. The table below is filled with numbers following certain pattern. If this table contains exactly 400 numbers, find the number located at the 14th row, 14th column.

…	…	…	…	…	…
…	**91**	**78**	**66**	**55**	…
…	**105**	**6**	**3**	**45**	…
…	**120**	**10**	**1**	**36**	…
…	**136**	**15**	**21**	**28**	…
…	…	…	…	…	…

【Solution】946 $(=43\times 44\div 2)$

Each number is in the form of $\dfrac{k\times(k+1)}{2}$.

The middle 2×2 square, has its left bottom corner as $\dfrac{4\times 5}{2}=\dfrac{2^2\times(2^2+1)}{2}$. (row 11, column 10)

The middle 4×4 square, has its left bottom corner as $\dfrac{16\times 17}{2}=\dfrac{4^2\times(4^2+1)}{2}$.(row 12, column 9)

The middle 6×6 square, has its left bottom corner as $\dfrac{36\times 37}{2}=\dfrac{6^2\times(6^2+1)}{2}$.(row 13, column 8)

From this number, go down 1 square and then go from column 8 to column 14, the required square is reached. 36+1+(14-8)=43. The required square is $\dfrac{43\times 44}{2}=946$.

19. Find the total number of natural numbers from 1 to 2023 that is either a multiple of 7 or is not a multiple of 3.

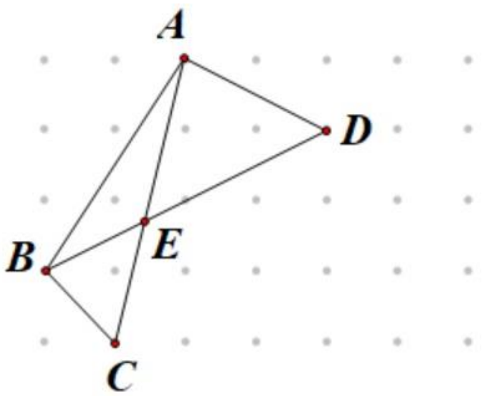
【Solution】1445

①Multiple of 7: $2023\times\frac{1}{7}=289$ ②Not multiple of 3: $2022\times\frac{2}{3}+1=1349$

Repeated count (multiple of 7 and not multiple of 3): $(289-1)\times\frac{2}{3}+1=193$

The answer: $289+1349-193=1445$

20. In the grid paper below, the side of each unit square is 1. If the ratio of BE to DE is $\frac{a}{b}$, which is a fraction in the simplest form, find the value of $a+b$.



【Solution】14

$BE:DE=\triangle ABC:\triangle ADC$

Method 1: Both areas can be calculated using Pick's Theorem

$\triangle ABC=2+\frac{3}{2}-1=2.5$ $\triangle ADC=3+\frac{5}{2}-1=4.5$

Method 2: The areas of $\triangle ABC$ and $\triangle ADC$ can be calculated as the big rectangle minus 3 triangles at the corner.

$\triangle ABC=4\times2-\frac{1}{2}\times3\times2-\frac{1}{2}\times1\times1-\frac{1}{2}\times4\times1=2.5$

$\triangle ADC=4\times3-\frac{1}{2}\times4\times1-\frac{1}{2}\times2\times1-\frac{1}{2}\times3\times3=4.5$

The area ratio is 5:9.

$a=5$ and $b=9$, and hence $a+b=14$.