2023 Spring Cup
Mathematical Olympiad
PRELIMINARY ROUND

Date： 28 January 2023

Time Given： 1 hour 30 minutes

Level: Primary 5

Name: _____

Instructions to Candidates

1. Do not open the booklet until you are told to do so.
2. Answer ALL 20 questions.
3. Write your answers in the answer sheet provided.
4. No steps are needed to justify your answers.
5. Questions 1-7 are worth 4 marks each.
6. Questions 8-14 are worth 6 marks each.
7. Questions 15-19 are worth 8 marks each.
8. Question 20 is worth 10 marks.
9. No marks will be deducted for wrong answers.
10. No marks will be given for unanswered questions.
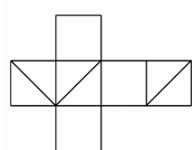11. No calculators or mathematical instruments are allowed.

<u>Questions 1 to 7 are worth 4 marks each.</u>

1. Calculate: $\dfrac{13}{35} \times 5 + \dfrac{13}{35} \times 6 + \dfrac{13}{35} \times 7 + \dfrac{13}{35} \times 8 + \dfrac{13}{35} \times 9$
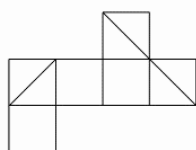
2. The sum of 7 natural numbers is 210. By arranging them in ascending order, the difference between each pair of adjacent numbers is 5, what is the 6th number?

3. It takes 225 digits to number the pages of a book, how many pages are there in this book?
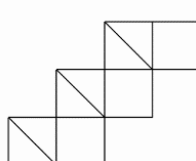
4. The "net" of a three-dimensional shape refers to its two-dimensional figure when laid out flat. Which of the following "net" corresponds to the cube on the right?
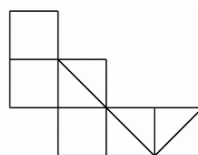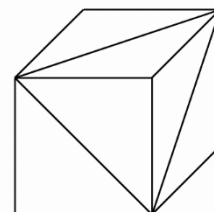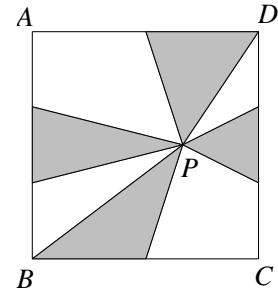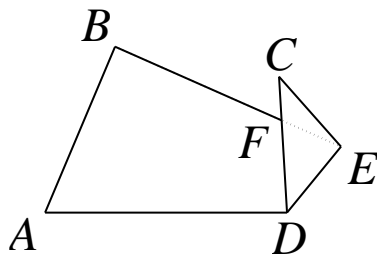

| A | B | C | D |

5. As shown in the figure, the square $ABCD$ has side 6cm and point P is a point lying in this square. If the sides of the square $ABCD$ are divided into two or three segments evenly such that they can form triangles with the point P, find the area of the shaded region.

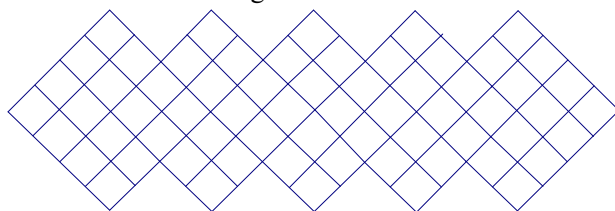6. Worker A takes 20 days to finish a job. Worker B takes 15 days to finish the same job. If worker A does the job for a few days before worker B takes over the job, it takes 18 days to finish the job. How many days does worker A and B work in this job respectively?

7. A triangle is folded along the line DE as shown below. Given that $\angle B = 74°$, $\angle A = 70°$ and $\angle CEB = 20°$, what is $\angle ADC$?

<u>Questions 8 to 14 are worth 6 marks each.</u>

8. How many squares can be found in the figure below?

9. Amelia and Benjamin want to purchase an item. According to the price tag, Amelia is short of 40 dollars and Benjamin is short of $\frac{1}{4}$ of the price. After negotiating, they received a 10% discount for the item, and they managed to make the purchase by combining their money. If they are left with 28 dollars, what was the original price of the item?

10. The ratio between red balls and white balls in a bag is $19:13$. First after adding in some red balls into the bag, the ratio becomes $5:3$. Then after adding in some white balls, the ratio becomes $13:11$. Given that the number of red balls added is 80 less than the number of white balls added, how many balls are there in the beginning?

11. It takes a train 52 seconds to go through a 320 meters long bridge. If the train speeds up by $\frac{1}{4}$ of its original speed, it can go across a bridge of 864 meters within 1 minutes and 36 seconds. Find the speed of the train when it goes across the bridge and the length of the train.

12. A total of 362880 distinct 9-digit numbers can be formed by using the number 1 to 9 without repetition. Find the greatest common divisor of the 362880 numbers.

13. Given the following vertical algorithm of multiplication between two numbers, if the blanks can only be filled with prime numbers, what is the two numbers that are being multiplied together? (Only write out the two numbers, not the product).
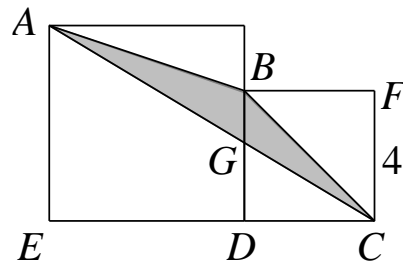

14. A 5-digit number gives different remainders when divided by 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. What is the largest possible value of this 5-digit number?

Questions 15 to 19 are worth 8 marks each.

15. Between the number 1000 to 1999 there are 1000 natural numbers, how many numbers are there in these 1000 numbers that have exactly two digits that are the same at the thousands, hundreds, tens and ones place?

16. The figure below is formed by a big square and a small square. If the side of the small square is 4cm, find the area of the shaded triangle ABC.

17. The distance between Jurong port and Pasir Panjang port is 360km, it takes a ferry 35 hours to travel back and forth these two ports and it takes 5 hours more to travel against the current than along the current. Now, there is a motor boat that can travel 12km/h in still water. How long does it would take for the motor boat to travel back and forth the two ports?

18. Given that a, b, c are three distinct numbers in 1 to 9. If a, b, c can form six 3-digit numbers and the sum of five of them is 2234, what is the other 3-digit number?

19. A natural number N can be written as the sum of 9 consecutive natural numbers, and can also be written as the sum of 10 consecutive natural numbers, and can also be written as the sum of 11 consecutive numbers, what is the smallest value of N?

Question 20 is worth 10 marks.

20. In your opinion, from question 1 to 19, your favourite question is question _____ and the most difficult question is question   .
(As long as your answer is within 1 to 19, you get full marks, otherwise you get zero.)