数学花园
探秘
MATHEMATICS GARDEN

2024 Spring Cup
Mathematical Olympiad

PRELIMINARY ROUND

Date: 28 January 2024

Time Given: 1 hour

Level: Primary 2&3

Name: _____

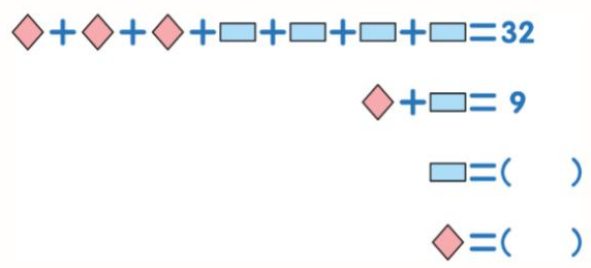
Instruction to Candidates

1. Do not open the booklet until you are told to do so.
2. Answer ALL 20 questions.
3. Write your answers in the answer sheet provided.
4. No steps are needed to justify your answers.
5. Questions 1-7 are worth 4 marks each.
6. Questions 8-14 are worth 6 marks each.
7. Questions 15-19 are worth 8 marks each.
8. Questions 20 is worth 10 marks.
9. No marks will be deducted for wrong answers.
10. No marks will be given for unanswered questions.
11. No calculators or mathematical instruments are allowed.

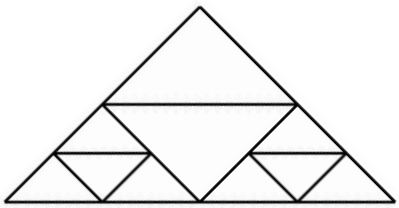
Questions 1 to 7 are worth 4 marks each.

1. $2-0\times2+4=$_____.

2. In the following calculations, the same shape represents the same number. Fill in the blanks.



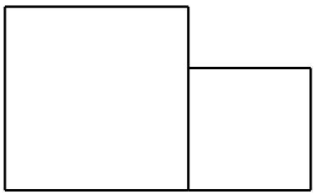
3. How many triangles are there?



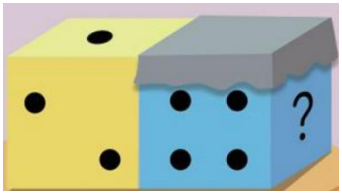
4. There are 3 groups of numbers below. Which 2 numbers can you swap such that when you sum the numbers in each group, you get the same answer?

5. $1\times9+2=11$
$12\times9+3=111$
$123\times9+4=1111$
.....
According to the above pattern, Δ and O are 2 numbers, $\Delta\times9+O=1111111$.
What is the sum of the digits in Δ and O ?
(For example: the sum of the digits in 123 and 4 is 1+2+3+4=10.)

6. Perimeter is the total length around the outside of a shape. The following figure consists of two squares, with the perimeter of the large square being 36cm and the perimeter of the small square being 20cm. What is the perimeter of this composite shape?



7. Each cube dice has six faces with 1, 2, 3, 4, 5, and 6 points, and the sum of the points on any two opposite faces is 7. The sum of the points on any two overlapping faces is 8. How many points are there on the face where the "?" is located?



Questions 8 to 14 are worth 6 marks each.

8. $1 \div 2 \times 2 \div 3 \times 3 \div 4 \times 4 \div \cdots \div 2022 \times 2022 \div 2023 \times 2023 \times 2024 =$ _____.

9. David and his friends went to the Mayday Noah's Ark concert. They spent a total of $2050 and bought a total of 11 tickets for CAT6 and CAT7. Among them, CAT6 tickets cost $200 per ticket, and CAT7 tickets cost $170 per ticket. How many CAT7 tickets did they buy?

10. If it is specified that numbers like 121, 6666 and 2442 that read the same from left to right and from right to left are called "palindromes". So, among all the 4-digit numbers, how many "palindromes" are there?

11. Alvin, Beryl, Celia and Derek are passing the ball, and Beryl always passes it to Celia every time. They have passed the ball 3 times. How many different passing situations are there?(For example, A→D→B→C, this is one passing situation)

12. The same letter represents the same number, while different letters represent different numbers. What is the four-digit number $\overline{ABCD}$?

$$\begin{array}{cccc} & C & D & C \\ + & A & B & C \\ \hline A & B & C & D \end{array}$$

13. Ada works part-time at a restaurant and her daily salary is the same. If she works for 30 days, she can buy a pair of shoes and have $240 left. After working for 25 days, she took her salary. After buying the shoes, she had $190 left. So how much do the shoes cost?

14. Some students lined up in a row. From left to right, Anna is the 7th student; From right to left, Brendan is the 8th student; There are two students between these two people. How many students are there in total? Write all the answers.

Questions 15 to 19 are worth 8 marks each.

15. 30 tigers and 30 foxes are divided into 20 groups, with 3 animals in each group. Tigers always tell the truth and foxes always tell lies. When asked if there are foxes in the group, 39 of these 60 animals answered “no”. So how many groups without foxes are there?

16. Four rectangles form a shape, and the length of each side of the rectangle is uncertain, so the shape of the entire shape is also uncertain. For example, Figure 1 and figure 2 are possible shapes. However, no matter how the length of each side changes, the perimeter of the rectangle *ABED* is always 15cm, and the perimeter of the rectangle *EFIH* is always 25cm. What is the maximum area of this entire shape?

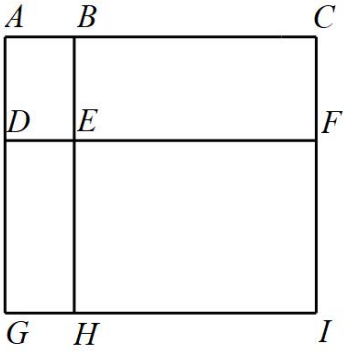


Figure 1

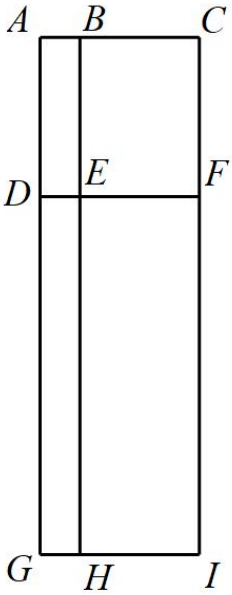


Figure 2

17. Select 9 numbers from the 10 numbers from 0 to 9 without repetition and fill them in the boxes in the following figure, so that the equation holds. What is the maximum four-digit number in the following figure?

□□+□□□=□□□□

18. Choose a non-zero whole number, if it is even, divide it by 2. If it is odd, multiply it by 3 and add 1. So we can get a new whole number. If we keep using this method, many of them will eventually become 1. For example: 5 is an odd number, so $5\times3+1=16$, 16 is an even number, so $16\div2=8$, $8\div2=4$, $4\div2=2$, $2\div2=1$. So, 5 needs to go through 5 times to become 1 for the first time. What is the sum of all the numbers that needs 10 times to become 1 for the first time?

19. Fill in an arrow in each blank cell at the edge, and the number in the square represents the number of arrows pointing to that number. The direction of the arrow can be up, down, left, right, top left, bottom left, top right, or bottom right, but each arrow points to at least one number. For example, the filling method in Figure 2 is the answer to Figure 1. Please fill in the arrow in Figure 3 according to this rule. How many arrows are pointing to the bottom right direction?

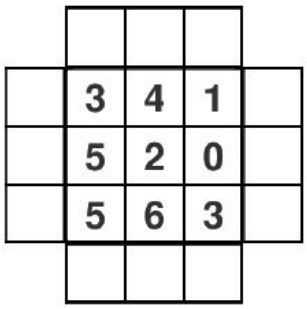


Figure 1

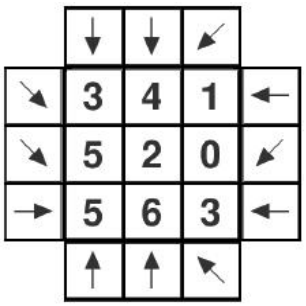


Figure 2

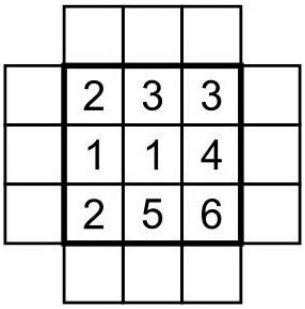
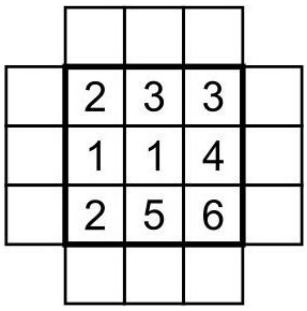
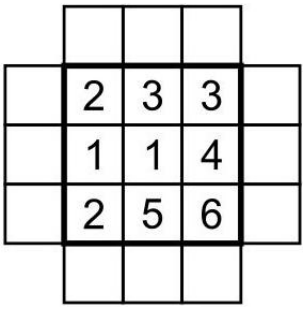
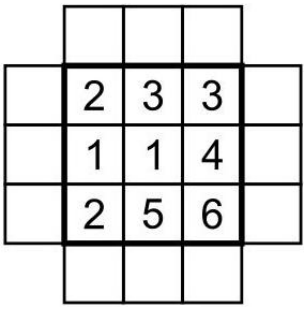


Figure 3

If necessary, you can use the following Figure 3:







Questions 20 is worth 10 marks.

20. In your opinion, from question 1 to 19, your favourite question is question _____, the most difficult question is question _____. (As long as your answer is within 1 to 19, you get full marks, otherwise you get zero.)