

# Singapore Mathematical Society

## Singapore Mathematical Olympiad (SMO) 2024

(Junior Section, Round 2)

Saturday, 22 June 2024

0900-1200

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### INSTRUCTIONS TO CONTESTANTS

1. Answer ALL 5 questions.
  2. Show all the steps in your working.
  3. Each question carries 10 marks.
  4. No calculators are allowed.
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1. Let  $ABC$  be an isosceles right-angled triangle of area 1. Find the length of the shortest segment that divides the triangle into 2 parts of equal area.
2. Let  $ABCD$  be a parallelogram and the points  $E, F$  are in the exterior. If triangles  $BCF$  and  $DEC$  are similar, i.e.,  $\triangle BCF \sim \triangle DEC$ , prove that triangle  $AEF$  is similar to these two triangles.
3. Seven triangles of area 7 lie inside a square of area 27. Prove that among the 7 triangles there are 2 that intersect in a region of area not less than 1.
4. Suppose for some positive integer  $n$ , the numbers  $2^n$  and  $5^n$  have equal first digit. What are the possible values of this first digit?
5. Find all integer solutions of the equation

$$y^2 + 2y = x^4 + 20x^3 + 104x^2 + 40x + 2003.$$