

Singapore Mathematical Society

Singapore Mathematical Olympiad (SMO) 2024

(Junior Section, Round 2)

Saturday, 22 June 2024

0900-1200

INSTRUCTIONS TO CONTESTANTS

1. Answer *ALL* 5 questions.
 2. Show all the steps in your working.
 3. Each question carries 10 marks.
 4. No calculators are allowed.
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1. Let ABC be an isosceles right-angled triangle of area 1. Find the length of the shortest segment that divides the triangle into 2 parts of equal area.
2. Let $ABCD$ be a parallelogram and the points E, F are in the exterior. If triangles BCF and DEC are similar, i.e., $\triangle BCF \sim \triangle DEC$, prove that triangle AEF is similar to these two triangles.
3. Seven triangles of area 7 lie inside a square of area 27. Prove that among the 7 triangles there are 2 that intersect in a region of area not less than 1.
4. Suppose for some positive integer n , the numbers 2^n and 5^n have equal first digit. What are the possible values of this first digit?
5. Find all integer solutions of the equation

$$y^2 + 2y = x^4 + 20x^3 + 104x^2 + 40x + 2003.$$