Singapore Mathematical Society

Singapore Mathematical Olympiad (SMO) 2024

Junior Section (Round 1)

Tuesday, 28 May 2024   0930-1200 hrs

Instructions to contestants

1. Answer ALL 25 questions.

2. Enter your answers on the answer sheet provided.

3. For the multiple choice questions, enter your answer on the answer sheet by shading the bubble containing the letter (A, B, C, D or E) corresponding to the correct answer.

4. For the other short questions, write your answer on the answer sheet and shade the appropriate bubble below your answer.

5. No steps are needed to justify your answers,

6. Each question carries 1 mark.

7. No calculators are allowed.

PLEASE DO NOT TURN OVER UNTIL YOU ARE TOLD TO DO SO.

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Multiple Choice Questions

1. If $x^2 + 4x + 16 = 0$, what is the value of x^3?

   (A) 4　　　(B) 8　　　(C) 16　　　(D) 64　　　(E) 128

2. Let a be a real number that satisfies $-1 < a < 0$. Which of the following is true?

   (A) $\pi^a < \dfrac{1}{\pi} < \left(\dfrac{1}{\pi}\right)^a$

   (B) $\pi^a < \left(\dfrac{1}{\pi}\right)^a < \dfrac{1}{\pi}$

   (C) $\dfrac{1}{\pi} < \left(\dfrac{1}{\pi}\right)^a < \pi^a$

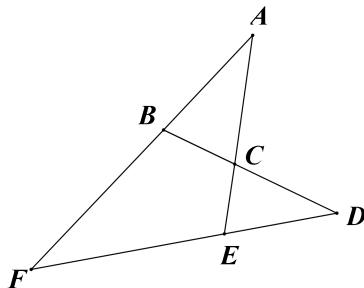   (D) $\dfrac{1}{\pi} < \pi^a < \left(\dfrac{1}{\pi}\right)^a$

   (E) $\left(\dfrac{1}{\pi}\right)^a < \pi^a < \dfrac{1}{\pi}$

3. How many non-congruent triangles are there whose sides have integer lengths and the longest side has length 10 units?

   (A) 25　　　(B) 30　　　(C) 35　　　(D) 40　　　(E) 45

4. In the diagram below, the points B and E lie on AF and DF respectively, and AE and BD intersect at C. If $AB = AC$, $BD = BF$ and $EA = EF$, find $\angle BAC$.

(A) $30°$ (B) $33°$ (C) $36°$ (D) $38°$ (E) $40°$

5. Let x, y and z be real numbers such that $x \neq 0$, $y - z \neq 0$ and $z + x \neq 0$. If $\dfrac{2}{x} = \dfrac{4}{y-z} = \dfrac{5}{z+x}$, find the value of $\dfrac{7x - y}{y + 2z}$.

(A) $\dfrac{11}{17}$ (B) $-\dfrac{11}{17}$ (C) $\dfrac{7}{13}$ (D) $-\dfrac{7}{13}$ (E) 1

6. Let N be a 2-digit whole number. When 2692 is divided by N, the remainder is 13, and when 2978 is divided by N, the remainder is 14. Find the sum of all the possible values of N.

7. If x is a positive integer such that $2^x + 2^{89} = 512^{10}$, find the value of x.

8. The diagram below shows a right-angled triangle ABC. The sides AC and AB are in the ratio 3 : 5. The point D lies on BC such that AD is perpendicular to BC. Furthermore, DB is 8 cm longer than CD. What is the length of BC in cm?
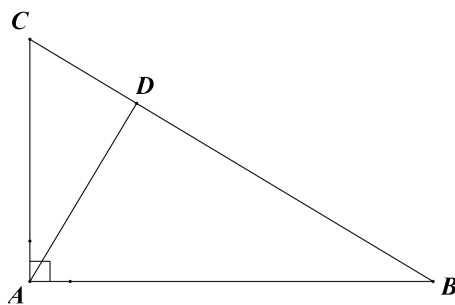
9. Let n be a positive integer. Suppose that a_1, a_2, a_3 $\cdots$ is a sequence of numbers defined by

$$a_1 = \sqrt{(n+3)(n-1)+4}, a_k = \sqrt{(n+2k+1)a_{k-1}+4} \quad \text{for } k \geq 2 .$$

If $a_{100} = 2024$, find the value of n.

10. Let N be the smallest positive integer such that the sum of its digits is 2024. What is the sum of the digits of the number $N + 2$?

11. If a and b are nonzero real numbers such that $\dfrac{1}{b} - \dfrac{1}{a} = 4$, find the value of $\dfrac{3a + 7ab - 3b}{a - 3ab - b}$.

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12. If the 5-digit whole number $\overline{11ab6}$ is a perfect square, find the value of $a+b$.

13. Let $N = 1 \times 2 + 2 \times 3 + 3 \times 4 + \ldots + 30 \times 31$. Find the value of N.

14. Let x and y be nonzero real numbers where $x \neq y$. If $x^2 + \sqrt{3}y = 4$ and $y^2 + \sqrt{3}x = 4$, find the value of $\left(\dfrac{y}{x}\right)^2 + \left(\dfrac{x}{y}\right)^2$.

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15. Find the sum of all 2-digit even numbers N with the following property: N is a multiple of the product of its two digits.

16. In the diagram below, AEB is an isosceles right-angled triangle and ABG is a $30° - 60° - 90°$ right-angled triangle with $\angle GAB = 30°$. The sides AG and BE intersect at H. If the area of triangle AHE is $50\,\text{cm}^2$, find the area of triangle BGH in cm^2.

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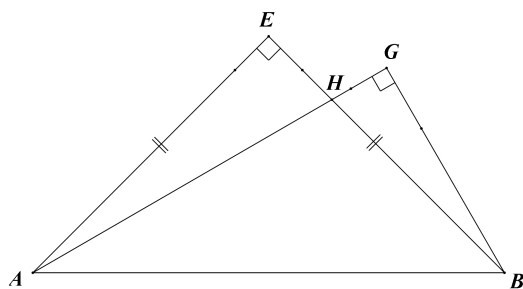
17. Find the smallest positive integer n such that $\sqrt{n} - \sqrt{n-1} < \dfrac{1}{99}$.

18. Let a, b and c be real numbers such that $a + b + c = 8$ and $ab + bc + ca = 0$. Find the maximum value of $3(a+b)$.

19. In the table below, every row and every column is an infinite arithmetic progression.

1	3	5	7	9	$\cdots$
3	6	9	12	15	$\cdots$
5	9	13	17	21	$\cdots$
7	12	17	22	27	$\cdots$
9	15	21	27	33	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\cdots$

How many times does the number 2025 appear in the table?

20. In the diagram below, ABC is a right-angled triangle. Points D and E lie on AB while points F and G lie on BC such that $\triangle EFG$ and $\triangle DGC$ are right-angled isosceles triangles. It is given that $DC = 3EG$ and the area of $\triangle DGC = 1$ cm^2. What is the area of $\triangle ADC$ in cm^2?

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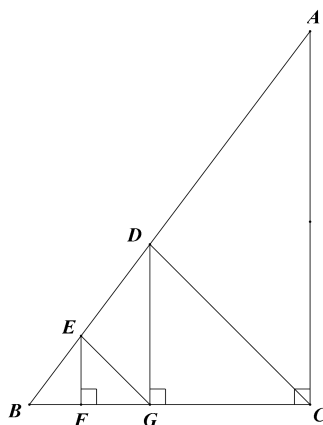
21. How many different 4-tuples (a,b,c,d) are there, where a,b,c and d are positive integers, such that
$$a > b > c > d, a + b + c + d = 2024 \text{ and } a^2 - b^2 + c^2 - d^2 = 2024?$$

22. Points A and B lie on the graph of $y = x^2 + 5x - 8$ such that A, B and the origin O are collinear and $|OB| = 2|OA|$. It is given that A lies in the first quadrant. Find $|AB|^2$.

23. The diagram below shows a regular hexagon that is divided into six congruent triangular regions A, B, C, D, E and F. Two triangular regions are adjacent if they share a common side. For example, A and B are adjacent but A and C are not adjacent. In how many ways can we colour these regions A, B, C, D, E and F using six different colours such that adjacent regions do not receive the same colour? (Note that not all six colours need to be used in colouring the six regions and non-adjacent regions can receive the same colour.)
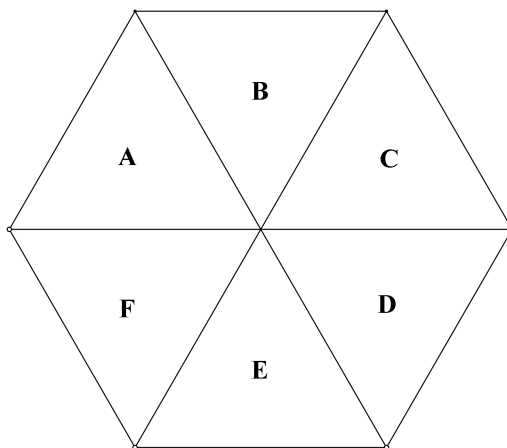
24. The diagram below shows three toy cars moving in a rectangular circuit $ABCD$ where $AB = 10$ m and $BC = 20$ m. Toy cars M and N start from the vertices A and C respectively and move in an anti-clockwise direction with constant speeds 10 m/min and 4 m/min respectively. Toy car P starts from C and moves in a clockwise direction with a constant speed of 8 m/min. The three toy cars start their motion at the same time. Assume that when any two toy cars meet, there is no collision and the cars will continue with their motion. Find the total time elapsed in minutes when all the three toy cars meet simultaneously for the fifth time.
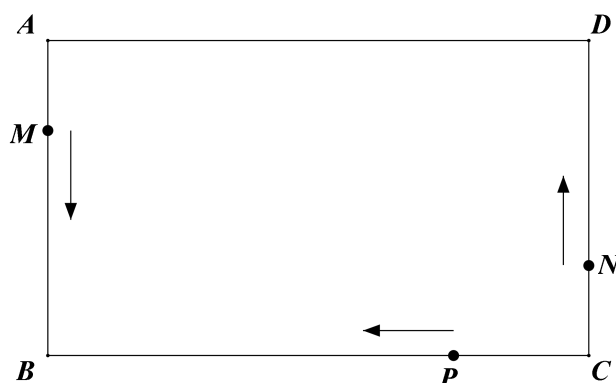
25. In the diagram below, *ABCD* is a rectangle. A circle of radius 240 mm is inscribed in △*ABD* and *BD* is a common tangent to both the circle and a semicircle whose diameter *CE* lies on *CD*. It is given that *CE* = 720 mm. Find the perimeter of rectangle *ABCD* in mm.